A further discussion on the effective thermal conductivity of metal foam: an improved model

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Abstract

In this study, we explain the causes and effects of the geometrical impossible result encountered in the widely adopted tetrakaidecahedron model (Boomsma and Poulikakos, 2001; Dai *et al.*, 2010) for the effective thermal conductivities (ETCs) of metal foam. The geometrical impossible result is successfully eliminated by accounting for the size variation of the node with porosity. The improved model provides predictions of ETCs that are more precise than available models. For aluminum foams ($k_s = 218$ W m^{-1}K^{-1}) using water and air as fluid media, the relative root-mean-square (RMS) deviation of the present predictions from the experimental data is about 5.3%; for the reticulated vitreous carbon (RVC) foams ($k_s = 8.5$ W m^{-1}K^{-1}), the relative RMS deviation is about 7.4%.

Key words: effective thermal conductivity; foam structure; node size; deviation; improved model.

Nomenclature

a	foam ligament radius (m)	L	ligament length (m)
d	dimensionless foam ligament radius	R	thermal resistance (m^2 K W^{-1})
e	dimensionless cubic node length	r	cubic node length (m)
f	function	V	volume (m^3)
k	thermal conductivity (W m^{-1} K^{-1})		

Subscripts

A	unit cell layer	D	unit cell layer
B	unit cell layer	eff	effective
C	unit cell layer	f	fluid

i	A, B, C, D	s	solid

Greek symbol

ε	porosity

1. Introduction

High-porosity metal foams are promising materials for thermal management applications. Since the effective thermal conductivity (ETC) is one of their most important thermal properties, an accurate evaluation of it becomes especially important. Studies on modeling the ETC of metal foams have been carried out numerically [1-3], experimentally [4-6] and analytically [5-12]. Among these approaches, the analytical approaches are less time consuming but more universal, and have attracted the attention of investigators. A review of the analytical approaches for prediction of the ETC has been conducted by Coquard and Baillis [13, 14], and Randrianalisoa and Baillis [15].

One of the most widely used analytical approaches was developed by Boomsma and Poulikakos [10] who first used the idealized three dimensional tetrakaidecahedron model to predict the metal foam ETC. Predictions were reported to accurately match the experimental data. However, Dai *et al.* [11] pointed out a few problems in their work. Dai *et al.* [11] extended the model by accounting for the ligament orientation. Predictions of the extended model were compared with the experimental data [5], and a relative RMS deviation of about 12% was observed. The deviation indicated that there was still room for improvement. In addition, results obtained in Ref. [11] showed that, as the porosity decreased, the diameter of the ligament became longer than the length of the node, which leaded to a geometrical impossible result. The diameter of the ligament should be shorter than the length of the node (see Fig. 1), which was a basic assumption in the development of the model.

In this paper, the tetrakaidecahedron model originally proposed by Boomsma and Poulikakos [10] and later extended by Dai *et al.* [11] is first discussed. The causes and effects of the geometrical impossible results are examined and explained. The model is further improved by accounting for the size variation of the nodes. We then show that the geometrical impossible results are eliminated. Lastly, predictions of our improved model are compared with several other analytical solutions as well as experimental data available in literature. It is shown that the current model has a steadily high precision in predicting the ETC of high porosity foams with a wide range of phase conductivity ratios (k_s / k_f).

2. Calculation of the Effective Thermal Conductivity

It is important to note that, for a better understanding of the present discussion, reader should be familiar with the analytical approaches developed by Boomsma and Poulikakos [10], and Dai *et al.* [11]. Therefore, in this part of the discussion, we give a brief review of how the ETC is calculated using their approaches. For more detailed discussions, reader may refer to Refs. [10, 11].

2.1. The Unit Cell of the Tetrakaidecahedron Model

Cubic nodes and cylindrical ligaments were used to represent the actual components of the foam network, as is shown in Fig. 1(a). As the lump shape at the ligament intersection varies with the foam porosity, the simplified spherical geometry is adopted for various porosities based on the fact that the lump volume has more significant effect on ETC than its shape does. The length of the node is r , the radius of the ligament is a and its length is L (from node center to node center). Based on the symmetry of the idealized model and one-dimension heat conduction along the z axis, a representative unit cell which contains all geometrical characteristics of the tetrakaidecahedron model was selected. The height of the unit cell in the z direction is $L\sqrt{2}/2$.While the length of the other two sides in the $x-y$ plane are both $L\sqrt{2}$. It can be proved that the ETC of the selected unit cell is equal to that of the tetrakaidecahedron model.

2.2. Effective Thermal Conductivity

In order to calculate the ETC, the unit cell is divided into four distinctive vertical layers along the z axis, namely A, B, C and D, as is shown in Fig. 1(b). The heights of the four layers are: $L_{\mathrm{A}} = a$, $L_{\mathrm{B}} = r/2 - a$, $L_{\mathrm{C}} = L\sqrt{2}/2 - r$ and $L_{\mathrm{D}} = r/2$.

According to the extend model (accounting for the ligament orientation) proposed by Dai *et al.* [11], the thermal resistance of each layer is

$$R_{\mathrm{A}} = \frac{4dL}{[2e^2 + \pi d(1-e)]k_{\mathrm{s}} + \{4 - [2e^2 + \pi d(1-e)]\}k_{\mathrm{f}}} \tag{1a}$$

$$R_{\mathrm{B}} = \frac{(e-2d)L}{e^2 k_{\mathrm{s}} + (2-e^2)k_{\mathrm{f}}} \tag{1b}$$

$$R_{\mathrm{C}} = \frac{2(\sqrt{2}-2e)L}{\pi d^2 k_{\mathrm{s}}\sqrt{2} + 2(2-\pi d^2\sqrt{2})k_{\mathrm{f}}} \tag{1c}$$

$$R_{\mathrm{D}} = \frac{2eL}{e^2 k_{\mathrm{s}} + (4-e^2)k_{\mathrm{f}}} \tag{1d}$$

where k_{s} is the thermal conductivity of the solid and k_{f} is thermal conductivities of the fluid, d and e are non-dimensional parameters, defined as: $d \equiv a/L$ and $e \equiv r/L$.

The overall thermal conductivity is calculated by assuming that the thermal resistances of the layers are connected in series. Based on the Fourier law of heat conduction, the ETC can be written as

$$k_{\mathrm{eff}} = \frac{L_{\mathrm{A}} + L_{\mathrm{B}} + L_{\mathrm{C}} + L_{\mathrm{D}}}{R_{\mathrm{A}} + R_{\mathrm{B}} + R_{\mathrm{C}} + R_{\mathrm{D}}} \tag{2}$$

Eq. (2) can be written as

$$k_{\mathrm{eff}} = f_1(d,e) \tag{3}$$

here, f_1 is a known function.

The porosity ε, which is defined as the ratio of the solid volume to the total volume can be calculate based on d and e as

$$\varepsilon = 1 - \frac{\sqrt{2}}{2}[de^2 + \frac{\pi d^2}{2}(1-e) + (\frac{e}{2} - d)e^2 + \pi d^2(1 - e\sqrt{2}) + \frac{e^3}{4}] \tag{4}$$

Solving for d in Eq. (4) gives

$$d = \left[\frac{\sqrt{2}(2 - 2\varepsilon - \frac{3\sqrt{2}}{4}e^3)}{\pi(3 - e - 2e\sqrt{2})} \right]^{\frac{1}{2}} \tag{5}$$

Substituting Eq. (5) into Eq. (3) gives

$$k_{\text{eff}} = f_2(\varepsilon, e) \tag{6}$$

where f_2 is another known function. Here, once the value of e is given, the ETC can be calculated purely by porosity.

3. Improved Model

We present a discussion on the model of Dai *et al.* [11]; highlighting the possible area of improvement. We then discuss the reason for the appearance of the geometrical impossible result. Our proposed model is then presented.

3.1. Revisit Dai et al.'s Model

<u>Precision:</u> In order to use Eq. (6) to predict the ETC, the value of e should first be calibrated. According to Ref. [11], a value of $e = 0.198$ was found to minimize the relative RMS deviation of predictions from the experimental data [5]. This deviation is about 12%, which indicates that there is still room for improvement.

<u>The Geometrical Impossible Result:</u> It should be true that $r/a > 2$ to ensure that the length of the node is larger than the diameter of the ligament. However, as has been mentioned in Ref. [11], this requirement can hardly be fulfilled with e = 0.198. The reason for the appearance of this

geometrical impossible result is explained next.

3.2. Causes and Effects of the geometrical impossible results

According to Eq. (4) or the structure of the tetrakaidecahedron model (see Figs. 1(a), 1(b)), a decrease in porosity ε, is attributed to an increase in *d* when *e* is held constant and vice-versa. Here, *d* and *e* can be considered to represent the diameter of the ligament and the length of the node. In the model of Boomsma and Poulikakos [10] and Dai *et al.* [11], the parameter e was set as a constant value. Therefore, according to Eq. (4), a decrease in porosity can only be realized by an increase in the diameter of the ligament *d*. As a result, as the porosity decreases, the diameter of the ligament increases while the length of the node remains constant, and eventually, the former exceeds the latter, leading to geometrical impossible results. In fact, the smaller the value of e, the more likely the geometrical impossible result occurs. As a result, geometrical impossible results were encountered more frequently in Ref. [11] than in Ref. [10].

3.3. Current Model

We improve on the model of Dai *et al.* [10] and eliminate the geometrical impossible result by accounting for the changing foam structure with porosity through the variable *e*. Since the experimental data contains information of the foam structure, using the experimental data, we can find how e varies with porosity. For a given porosity, we calibrate the e value by comparing the predictions made by Eq. (6) against experimental data (Ref. [5]). As a result, values of e for ten given porosities are obtained, as is shown in Fig. 2. The parameter *e* can be fitted by a third order polynomial function of the porosity ε (Fig. 2) as

$$e = a + b\varepsilon + c\varepsilon^2 + d\varepsilon^3 \quad (7)$$

where *a* = 327.25811, *b* = -1075.55645, *c* = 1182.83207 and *d* = -434.55535. The fitting error is less than 1%.

4. Model Validation

After obtaining the function between e and porosity, the ETC can be predicted as a function of porosity. Substituting Eq. (7) into Eq. (6), gives:

$$k_{\text{eff}} = f(\varepsilon) \tag{8}$$

where f is a known function.

When Eq. (8) is used to compute the ETCs for the experimental data in Ref. [5], the relative RMS deviation is about 5.0% for water-saturated foams, and 5.6% for air-saturated foams. The ratios of the node length to the ligament radius are also predicted using the current approach. As a result, within the porosity of 0.905 to 0.978, the r/a ratios are always > 2.0 (decreasing from 6.33 to 2.71). Thus, the geometrical impossible results are eliminated.

Figure 3 shows comparisons of our model with selected models. As a result of the changing e values with porosity, our model is capable of capturing the non-linear variations in the ETCs as function of porosity. The relative RMS deviations from water and air experimental data of the present, Yang's, Dai's and Paek's predictions are 5.3%, 11.7%, 13.2% and 13.3% respectively. Since the e value is calibrated from the experimental data in Ref. [5], the high precision against the experimental data in Ref. [5] is expected. To validate our model, we use Eq. (8) to compute the ETCs for the experimental data reported by Phanikumar and Mahajan [16]. They reported the ETCs of air-saturated Al foams with porosity ranging from 0.899 to 0.959. Due to a similar foam geometry, the present model is capable of accurately predicting the variation trend of the ETCs with porosity; with a relative RMS deviation of about 12.1%.

To further assess the validity of our improved model as well as the fitted e value (Eq. 7).

We use our model to predict the ETCs of high porosity RVC foams (which have similar structure with high porosity metal foams), and the results are compared with experimental measurements reported in Ref. [6]. As a result, for water-saturated RVC foams, all the analytical models can accurately predict the ETCs (RMS deviation < 10%). When it comes to air-saturated RVC foams, Yang's model and Peak's model are less accurate with a RMS deviation of more than 21.8%. The relative RMS deviations of the present model are relative small; 7.2% and 7.6% for water-saturated and air-saturated RVC foams respectively. These results indicate that our improved model have a wider range of applicability.

5. Conclusions

We accounted for the size variation of the node with porosity and successfully eliminate the geometrical impossible results. The improved model provides more accurate predictions of the ETCs. Comparisons with other analytical models as well as experimental data validates that our model has a steadily high precision in predicting ETCs of foams with a wide range of solid phase to liquid phase conductivity ratios (k_s / k_f).

Conflict of interest

None.

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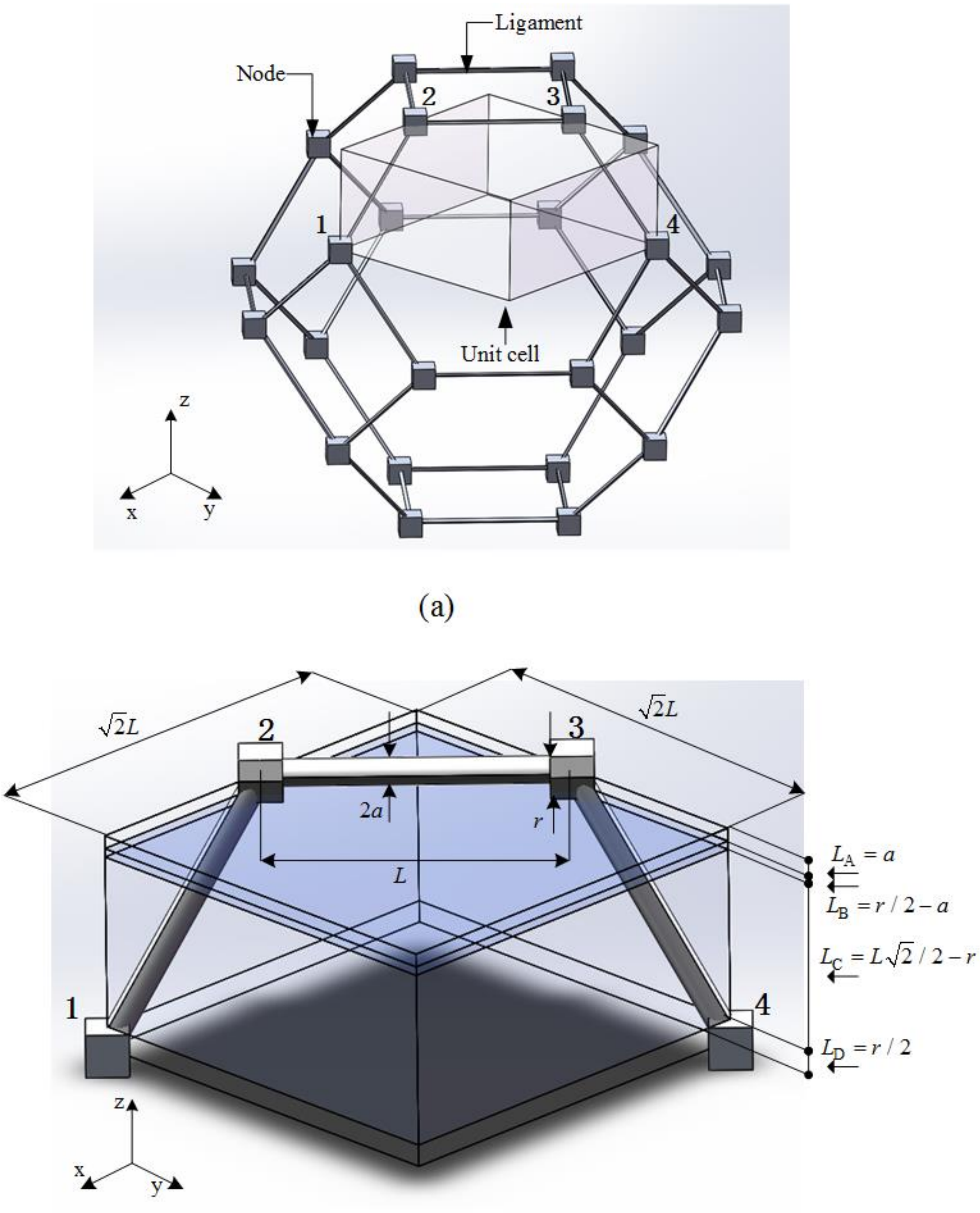


Fig. 1 (a) The tetrakaidecahedron model and (b) four distinctive layers for the unit cell

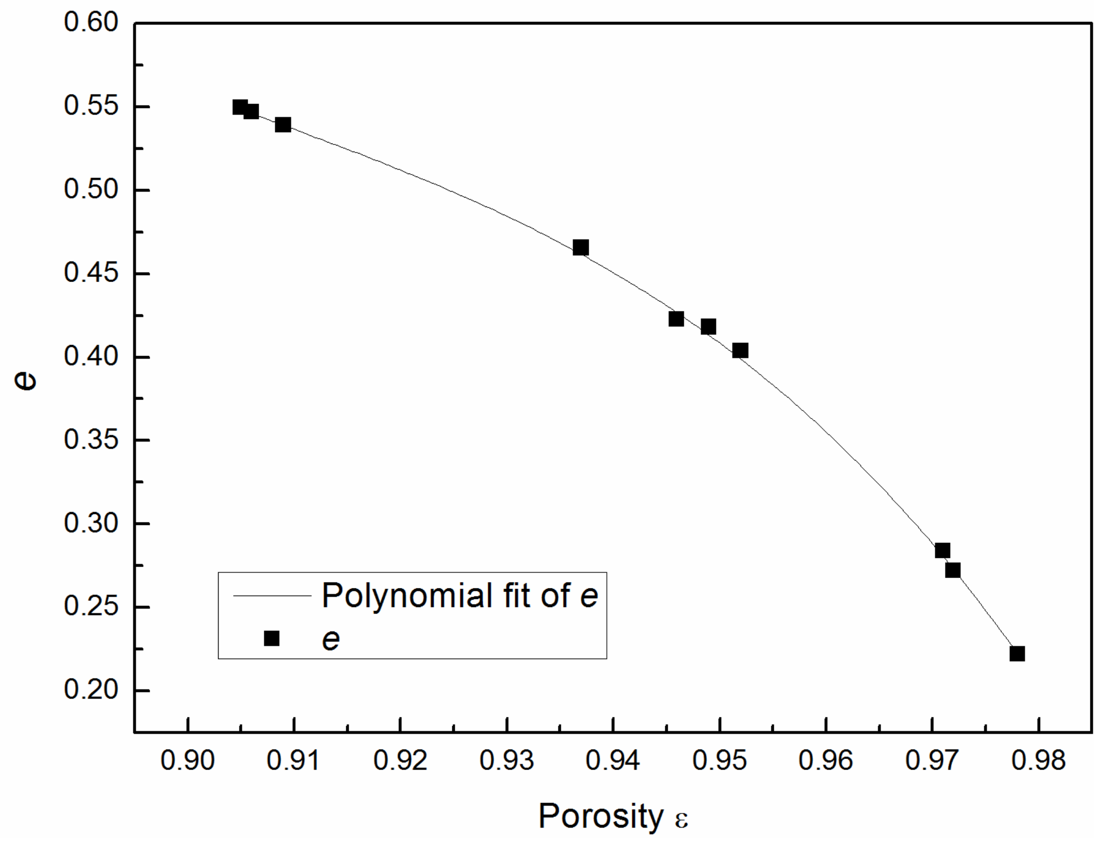
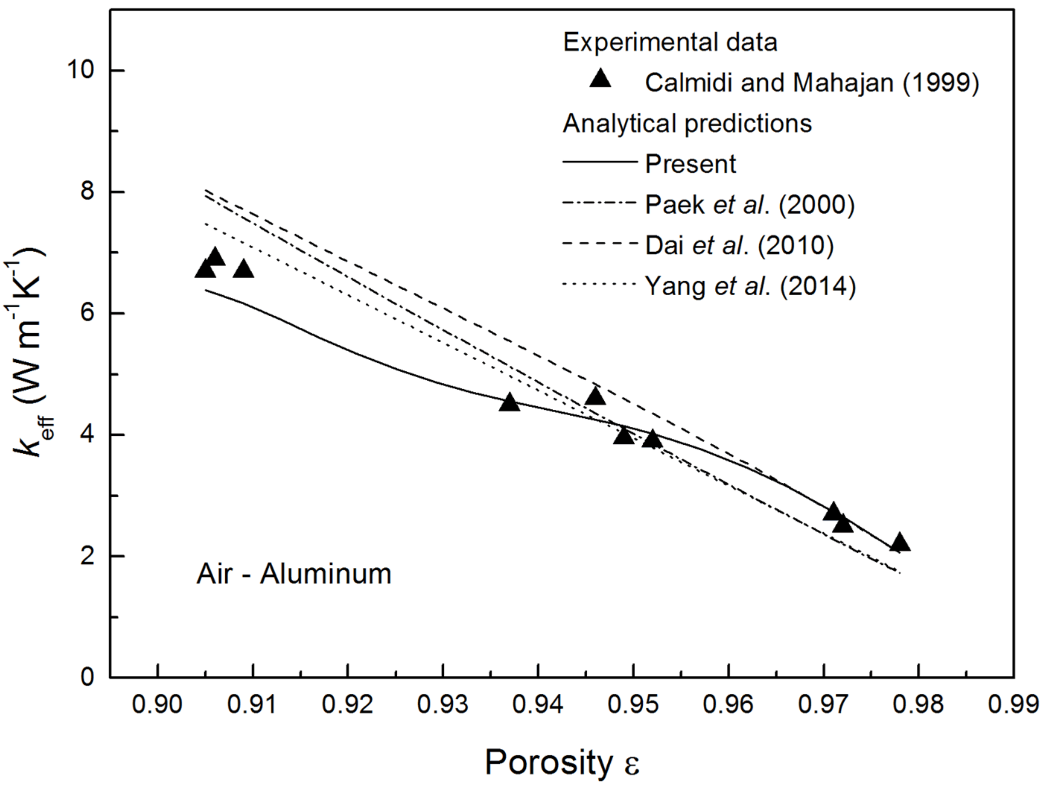
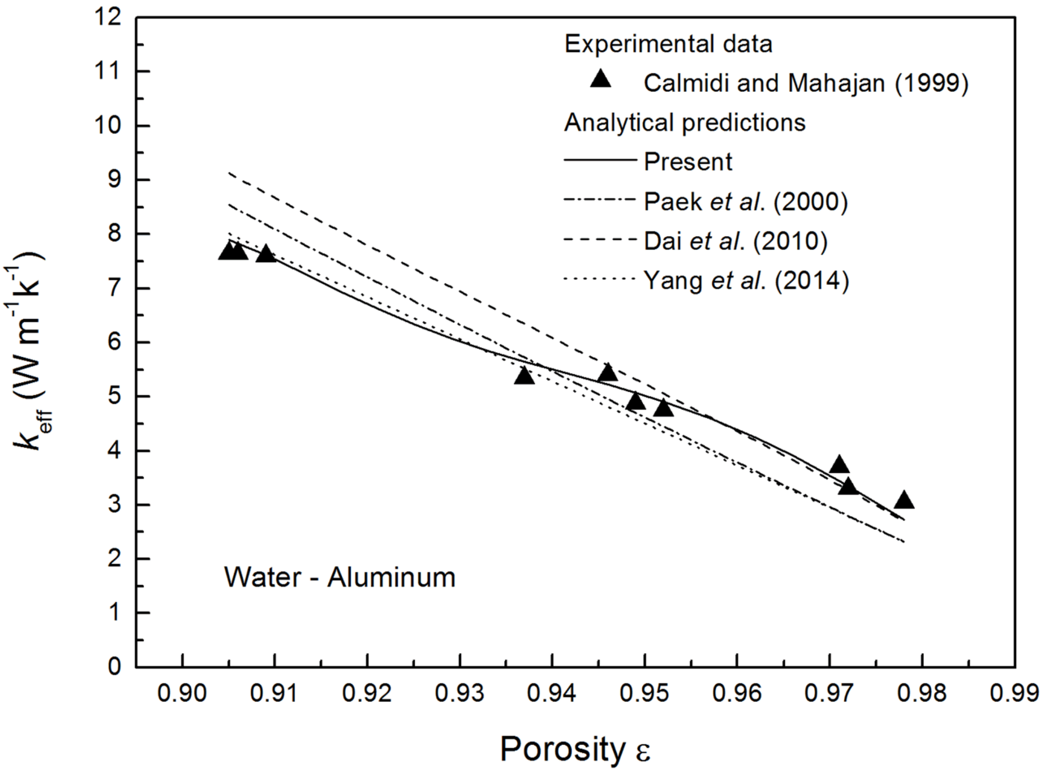


Fig. 2 Calibrated e values



(a)



(a)

Fig. 3 Comparisons between the analytical models and the experimental data of (a) air-aluminum and (b) water-aluminum