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Original Citation

Jan, Steven (2015) Memetic Perspectives on the Evolution of Tonal Systems. *Interdisciplinary Science Reviews*, 40 (2). pp. 145-167. ISSN 0308-0188

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MEMETIC PERSPECTIVES ON THE EVOLUTION OF TONAL SYSTEMS IN MUSIC

Abstract

Cohn (1996) and Taruskin (1985) consider the increasing prominence during the nineteenth century of harmonic progressions derived from the hexatonic and octatonic pitch collections respectively. This development is clearly evident in music of the third quarter of the century onwards and is a consequence of forces towards non-diatonic organization latent in earlier music. This article conceptualizes such forces as memetic – drawing a distinction between memetic processes in music itself and those in the realm of music theory – and interprets the gradualistic evolution of tonal systems as one of their most significant consequences. After outlining hypotheses for the mechanisms driving such evolution, it identifies a number of “musemes” implicated in hexatonic and octatonic organization in a passage from Mahler’s Symphony no. 10. Anthony Pople’s (2013) *Tonalities* music-analysis software is used to explore the tonal organization of the passage, which is considered in relation to the musemes hypothesized to generate and underpin it.

Keywords

Meme, museme, musemplex, tonality, *Tonalities*.

1. Introduction: Memetics and the Evolution of Tonal Systems

1.1. Hypothesis

My contention in this article is that *memetics* – a theory of cultural evolution predicated on a hypothesized analogy between cultural and biological “replicators”, memes and genes respectively (Dawkins, 1989; Blackmore, 1999) – offers an indispensable conceptual framework for the understanding of *tonal systems* in music. A tonal system represents the highest level of musical pitch organization, describing and prescribing the ground rules for the combinations of pitches in chords and chords in sequences or progressions. I assert that memetics is key in accounting for the variation, transmission and selection of the constituent elements of all tonal systems and can therefore explain the evolutionary changes which occur in those elements and, consequently, the system itself over time. Given this arguably bottom-up, emergent nature of tonal systems, the focus here is squarely upon the lowest level of structure, at the unit musical memes (hereafter “musemes”, a contraction of “musical memes”) whose attributes and alignments are responsible for the configuration and reconfiguration of the system. These musemes include, in the case of the classical “diatonic” major-minor tonality of 1700–1875 which forms the focus here, those which embody such tonality-defining phenomena as the scale-degree motions $\hat{7}-\hat{8}$ and $\hat{4}-\hat{3}$ and the root/bass motion $V/\hat{5}-I/\hat{1}$, all features associated with key-defining and closure-engendering cadential schemata (Gjerdingen, 2007).

1.2. Premises

Anthony Pople’s term “tonalities” was coined to convey a sense of the multiplicity of practice in harmonic language and tonal organization in music of the late-nineteenth and early-

twentieth centuries (Pople, 2004; Russ, 2004).¹ It neatly characterized a problem which was the impetus for his development of a sophisticated solution, the pitch-class set analytic software known as *Tonalities* (Pople, 2002, 2013). This was designed to map the differences between the output of a Debussy and a Schoenberg and in so doing to help formulate a “*tonal set theory*” (Pople, 2004, p. 155; his emphasis).² Pople’s is a useful concept because at this time in musical history divergent approaches to tonal organization arose in the work of contemporaneous composers in a way which did not occur a century earlier. Cumulatively, such lower-order, “idiom-” (composer) and “dialect-” (chronological and geographical style) level processes fed into and determined the complexion of higher-order tonal systems situated at the more intangible level of “rules” (Meyer, 1996, p. 23; Jan, 2013, p. 152, Figure 1).

Except when viewed from the perspective of a thin slice through time, a tonal system is in constant dynamic flux – this fluidity posing a fundamental challenge to the very concept of rules, with all its implications of immutability and permanence. As a diachronic reality, not a synchronic abstract, a tonal system exists concretely through the myriad figures and patterns which constitute the elements of the music and which are passed from pupil to teacher and from peer to peer. It follows that the various characteristics, qualities, tendencies and relationships of these figures and patterns give rise, as bottom-up forces, to the attributes and dynamics which constitute and define the system.

One way of regarding these “figures and patterns” is to see them as memes in music – as *phenotypic* manifestations of discrete neural constellations, *memotypes*, transmitted and reconstituted from brain to brain by imitation (Jan, 2007, 2011). It is axiomatic to memetics that large-scale cultural phenomena – including tonal systems – are manifestations of the attributes of “real” memes at lower levels of a cultural hierarchy feeding upwards to engender larger scale “virtual” networks and relationships at higher, systemic levels. In this view there is no central authority governing a tonal system, no omniscient arbiter of the realm. Such systems are thus macrocosms of human consciousness, some widely accepted views of which see it as a (virtual) artefact, the cumulative consequence of myriad low-level distributed processes of “continuous ‘editorial revision’” (Dennett, 1993, p. 111; Blackmore, 2010).

Because memetics is a theory of culture based on the transmission of discrete particles of information in a variety of substrates, it follows that the theoretical models which have arisen to describe tonal systems are also memes. They are *verbal-conceptual memes* which, like musemes (in *musemeplexes*), associate to form *memeplexes* (Jan, 2003, 2007, p. 80ff.) – an issue considered further in Section 2.1 below – articulating a particular perspective on music. Such memeplexes are subject to selection pressures comparable to those acting upon memes in other substrates, including the criterion of fit against that which they purport to model. While I shall focus here primarily on the evolution of musemes, a comprehensive treatment of this

¹ I am indebted to Michael Russ and the anonymous reviewers for their comments on earlier drafts of this article. Some passages here are adapted from (Jan, 2007).

² Pople (b. 1955) died prematurely in 2003. A complete reverse-engineering of *Tonalities* has not, at the time of writing, been attempted. See (Dunsby, 2004).

subject would require consideration of both memetic realms and their coevolutionary interaction.

In what follows, I outline, in Section 2, two fundamental mechanisms by which musemes are capable of driving tonal-systemic change. Section 3 relates these hypotheses to the evolution of certain disruptive phenomena in tonal-system organization – namely “hexatonic”, “octatonic” and “whole-tone” structures – which become increasingly evident around the middle of the nineteenth century, and it discusses how *Tonalities* might be used to understand them. Section 4 applies the necessarily abstract arguments of Sections 2 and 3 to a concrete example, considering a passage by Mahler which affords evidence both for the specific mechanisms of Section 2 and for the more general claim made in Section 1. Section 5 identifies some of the many avenues for future research the argument developed here opens up.

2. Mechanisms

Musemes exert their bottom-up, tonal-system (re)configuring power in two ways: by juxtaposition – that is, the combination of two or more established musemes in order to create a new configuration with novel internal tonal relationships – and by mutation – that is, by changing an established museme in ways which make it tonally novel.

2.1. Juxtaposition

Juxtaposition describes the consequences of (i) the sequential abutting/parataxis of independent musemes and/or (ii) their co-replication, “horizontally” or “vertically” aligned, in musemplexes. In the former case, while the constituent replicated units of a passage have their own internal coherence (largely defined by their gestalt-demarcating initial and terminal articulations), *en masse* they collectively define the pitch content and harmonic/tonal organization of a passage. In the latter case, even though works of music are seen in memetics as alliances or coalitions of essentially independent, selfish replicators, a given passage of music may nevertheless also contain a musemplex – a collection of musemes which, while its members have independent existence, is replicated in at least one other context as a group. Even a seemingly solitary museme is normally itself a multiparametric complex – a collection of potentially independent uniparametric musemes (Narmour’s “style forms/shapes”, the composite museme being a “style structure” (Narmour, 1977, pp. 173–4, 1990, p. 34)) – in the domains of pitch, rhythm, melody and harmony.

On this reasoning, musemes which while logical and coherent within the prevailing theoretical system as separate entities may generate new tonal structures as a result of novel sequential juxtapositions (scenario i above) and/or associations in musemplexes (scenario ii). Thus, some such alignments might give rise to a composite musemic-sequential structure M^1-M^2 which generates a shallow-middleground voice-leading structure or harmonic progression x , which is recognized and endorsed by theory. Others, however, might generate a composite M^3-M^4 which generates a voice-leading structure or progression y , which is neither recognized nor, therefore, so endorsed.

Recast in the terms of Pople’s *Tonalities* project, x represents a structure in which M^1 and M^2 intersect individually with one or more (closely related) diatonic *prolonging gamut(s)*, and which may be understood in terms of a single diatonic *connective* [i.e., overlapping, common]

gamut (usually the same as or closely related to the prolonging gamuts). A gamut is Pople's term for a pitch collection – essentially, a scale type. By contrast, and as considered further in Section 3.3, y represents a structure which, while M^3 and M^4 may still intersect individually with one or more (less closely related) diatonic prolonging gamut(s), is only understandable in terms of a *non*-diatonic connective gamut (Pople, 2004, pp. 164, 182). Thus, the new musemic-sequential structure, by virtue of its component elements and their relationship, creates a pitch aggregate which can only be understood by an appeal to a different (generally broader) memetic-theoretical paradigm.

2.2. Mutation

Mutation describes the consequences of configurational changes to individual musemes resulting from miscopying. If replicated widely, certain mutational changes generate – in a bottom-up musemic snowball effect – higher-order change within the dialect and then within the system of rules: they open up the syntactic possibilities of the system in ways which, while often baffling to contemporary theorists, are retrospectively assimilated and eventually normalized. Beyond a certain statistical prevalence, the sheer cultural weight of a mutant museme x^1 in a community renders it normal, even though, in the prevailing theoretical models of the time (which ossify the *status quo ante*), its antecedent x is seen as the (only) “legitimate” form.

Narmour's Implication-Realization model (Narmour, 1990, 1992) suggests that the realization of latent implications is one of the (potentially several) engines which drive musemic mutation. In a telling example, he argues that such simple patterns as rising melodies carry latent tendencies to further upward melodic expansion (Narmour, 1977, p. 129, Example 44). This may be attenuated by gestalt end-closure (via rests or long note endings), resulting in a notional “style” which only encompasses the unextended melody. But eventually the latent tendency will be realized because, almost unstoppably, it impels miscopying, and then the frontiers of the style will have been extended to encompass the mutant pattern, provided the latter is replicated. In short, when a museme mutates it is likely to do so in accordance with the dictates of its internal implication-realization structures (see also Jan, 2014, sec. 2.2).

Cohn argues that “[w]hen force is sufficient to cause the diatonic barriers that segregate [anti-diatonic pressures] to overflow, hexatonic triadic progressions begin to emerge” (1996, p. 31). He attributes this “force” to a trinity of factors: the use of consonant (i.e., major or minor) triads, “parsimonious” (zero or minimal-distance) voice-leading, and the preservation of common tones across two or more harmonies (1996, p. 31). I would assert – in the light of Cohn's concession that “the absolute integrity of diatonic collections has been under constant pressure from a *variety* of forces throughout musical history” (1996, p. 31; my emphasis) – that a fourth (but not necessarily final) force is exerted by the implication-realization pressure illustrated by the aforementioned example by Narmour, this acting between (juxtapositionally) and within (mutationally) musemes. This “I-R drive” offers a diachronic, melodic and psychologically grounded complement to the more synchronic, harmonic and abstractly mathematical “Neo-Riemannian”/transformational phenomena theorized by Cohn and others for the understanding of non-DIA progressions in music (Lewin, 1987; Cohn, 1997; Tymoczko, 2011).

3. Memetics and the Evolution of Diatonic, Hexatonic, Octatonic and Whole-Tone Collections

3.1. Pitch Collections and Structural-Hierarchical Levels

A significant tonal-systemic change in the music of the mid- to late-nineteenth century, one readily explicable in terms of the mechanisms discussed in Section 2, is the increasing prevalence of hexatonic, octatonic and whole-tone pitch collections (hereafter “HEX”, “OCT” and “WT”, respectively), expressed as melodic shapes and/or harmonic progressions. These challenge the supremacy of the prevailing diatonic (hereafter DIA) context orientated around the circle of fifths which had regulated most European music from the mid-Baroque period.³ Tracing the evolution of HEX, OCT and WT organization in music of the late-nineteenth century arising from the juxtaposition and mutation of antecedent DIA musemes would offer strong (if circumstantial) evidence in support of the claim made in Section 1.1.

Thus, in arguing in Section 1.1 that “memetics ... offers an indispensable conceptual framework for the understanding of tonal systems in music”, because it is “key in accounting for the variation, transmission and selection of the constituent elements of all tonal systems and can therefore explain the evolutionary changes which occur in those elements and, consequently, the system itself over time”, the “variations” with which I am principally concerned here are those which align (juxtapose) or deform (mutate) musemes and musemeplexes articulating predominantly DIA pitch collections into those capable of being seen as articulating HEX, OCT and WT collections.⁴ While beyond the scope of this article, transmission may be understood in terms of the differential *perceptual-cognitive salience* of the resulting variant patterns; and selection appears to be a function, in part, of the degree of fit, diachronic and synchronic, of variant patterns with others in the museme-pool.

In terms of Allen Forte’s pitch-class set theory (Forte, 1973), juxtapositions and mutations alter the set-class one or more musemes express such that, for instance, a DIA_{0,2} chord-progression museme I–vi [0,2,4,7,9] may become the HEX_{3,4} progression I–*b*VI [0,3,4,7,8]. Similarly, juxtaposition of musemes articulating DIA’s subsets, such as the major and minor triads (3–11 (037)), the diminished triad (3–10 (036)) and the augmented triad (3–12 (048)), gives rise to progressions characteristic variously of HEX, OCT and WT. All these chords (and the strictly non-DIA diminished seventh chord (4–28 (0368))) existed before HEX, OCT and WT became established as contrasting tonal systems to DIA, because chromaticisms were long naturalized within DIA contexts, often in the context of chromatic alterations to the “natural”

³ For an overview, see (Cohn, 1996, p. 24, Figure 5; Straus, 2005, pp. 140–50, 154–5). In short, HEX-influenced music draws upon such six-note scales as C–E_b–E_b–G–G_#–B; OCT-influenced music is based upon eight-note scales such as C–C_#–E_b–E_b–F_#–G–A–B_b; and WT-influenced music uses such six-note scales as C–D–E–F_#–G_#–A_#. DIA music uses seven-note scales which only employ the pitches specified in a “key-signature”, such as the C–D–E–F–G–A–B sequence constituting C major.

⁴ “Being seen as articulating” may involve a relatively drawn-out process. In the case of OCT, while understanding of the collection as a distinct compositional resource is arguably evident in the works of composers from the early-nineteenth century, the term itself was only coined by Arthur Berger in 1963 (as Gould and Vrba point out, “unnamed ideas generally remain unconsidered” (1982, p. 4)). See (Forte, 1991, pp. 125, 160, note 5; Taruskin, 1985, p. 79f.) for discussion of “proto-OCT” passages in Schubert and Beethoven.

(DIA) minor and their replication in the major. Yet the cumulative weight of such mutations ultimately brings about system-level change, such that DIA-orientated musemes relinquish their (statistical) hegemony and are forced to coexist with those articulating HEX, OCT and WT collections.

Even without our accepting all the theoretical claims of Schenkerian analysis (Schenker, 1979), it is generally acknowledged that in DIA music linear/scalic patterns – sometimes highly chromatic ones – often “prolong” deeper-level triadic subsets via passing or auxiliary motion. Developing this perspective further, memetics suggests that musemes are propagated at several structural-hierarchic levels, and that foreground-level musemes – which may exist as (i) single horizontal (triadic or linear) strata; (ii) agglomerations of such strata into contrapuntal-harmonic sequences (musemplexes); and (iii) triadic/harmonic progressions relatively autonomous of fixed contrapuntal voices – generate (and constitute the interstitial prolongations of) those at the shallow-middleground level, and so on recursively to the background. While such deeper-level musemes may come into any or all of the three categories just outlined, they often consist of essentially triadic sequences with associated linear progressions; this of course is one definition of the superordinate-level *Ursatz* (fundamental structure). It follows from this (and from orthodox Schenkerism) that DIA organization is strongest at deeper structural levels, succumbing last to the onslaught of musemic chromaticism which per/invaded first the foreground level and then the middleground.

3.2. HEX, OCT and WT via Musemic Juxtaposition and Mutation

I have suggested that HEX, OCT, and WT progressions are either the result of the juxtaposition of DIA-orientated musemes into patternings which engender HEX, OCT, or WT structures; or they are the result of the mutation of DIA-orientated musemes into forms which have the potential (justified and validated *post facto* in theory) for a HEX, OCT, or WT interpretation. Moreover, it seems likely that the foregoing order is indeed that which occurred in musical history. One might formalize this as the following two hypotheses, represented diagrammatically in Figure 1:

1. HEX, OCT and WT progressions *initially* appear at the foreground and shallow-middleground levels as the inter-museme result of the juxtaposition of DIA musemes in linear sequences and in musemplexes (Section 2.1).⁵ In this sense, HEX, OCT and WT organization are the consequence of *first*:
 - a) musemes being horizontally juxtaposed (·|·) in linear sequences and in musemplexes in ways which create localized foreground and shallow-middleground juxtapositions “across the cracks” (inter-museme horizontal juxtaposition, hereafter “IMHJ”); *and then...*

⁵ Note that some musemes consist of more than one stratum of music, so it is not appropriate, in their case, to speak of inter-museme vertical juxtaposition.

- b) musemes being vertically juxtaposed (\div) in musemeplexes in ways which create localized foreground and shallow-middleground HEX, OCT and WT collections (inter-museme vertical juxtaposition, hereafter “IMVJ”).

and then...

2. HEX, OCT and WT progressions also arise from intra-museme mutation of DIA musemes (hereafter “IMM”) (Section 2.2). Thus, as inter-museme (juxtapositional) HEX, OCT and WT organization became widely propagated, these collections were then directly generated – mimicked? – by intra-museme mutation.

Figure 1: Generation of HEX, OCT, and WT Collections

3.3. Using *Tonalities* to Analyse HEX, OCT and WT Collections

As suggested in Section 1.2, Pople’s *Tonalities* software is an analytical tool useful for understanding the different approaches to tonal organization in individual works and the works of individual composers which, in aggregate, constitute the tonal system of a particular dialect and which, in conjunction with chronologically and geographically adjacent dialects, define some system of rules.

In brief, *Tonalities* operates by assessing the pitch content of segments of music (encoded as Microsoft *Excel* spreadsheets), testing them against various *language settings* which determine which gamuts and chords it should report if present. In this sense, it is a highly sophisticated pitch-class set analysis tool capable of showing which harmonies underpin the music, which Fortean sets are prolonged by these harmonies, and which sets serve to connect prolonged sets, in any given segment (symbolized by “\$” in the spreadsheet) of music. To refine the brief discussion in Section 2.1, a prolonging gamut is a pitch collection within which a single harmony is prolonged, whereas a connective gamut is a link or bridge between two prolonging gamuts calculated by extracting a trichord (three-note chord) from two adjacent prolonging gamuts and finding the superset of which they are both subsets.

In terms of the hypotheses of Section 3.2, where IMHJ leads to adjacencies which, owing to their chromaticism, make the chord-to-chord (and museme-to-museme) continuity increasingly problematic from the perspective of a DIA-orientated theory, musemic interrelationships (and therefore the tonal structure) may often be best understood by invoking a HEX, OCT or WT connective gamut. Later, when IMHJ and IMM become more prevalent, HEX, OCT and WT collections become more securely established as prolonging (in addition to connective) gamuts.

Using its default language settings, *Tonalities* is insensitive to HEX, OCT and WT collections, attempting to read chromaticism from the perspective of DIA collections and, failing this, describing it (in a white-flag default) as CHR[omatic] (i.e., using a subset of western music’s ultimate 12-note superset). In this respect it is mimicking the behaviour of music theorists who try, and often fail, to see new phenomena through old spectacles. Sensitizing *Tonalities* to HEX, OCT and WT collections is equivalent to the inception and acceptance of a new theoretical paradigm – the propagation of a new verbal-conceptual memeplex – which permits and mediates an arguably more insightful view of the music.

4. A Case Study in the Evolution of HEX, OCT and WT Collections

4.1. Preliminaries

To offer evidence in support of the hypotheses in Section 3.2, I now turn to a passage from the Adagio of Mahler's Symphony no. 10 (1910).⁶ This extract is chosen because of Pople's and Russ's (implied) view that its tonal ambiguities make it both analytically interesting and theoretically problematic; because its component musemes are relatively easily identified (on both gestalt-psychological and coindexation-determined criteria (Jan, 2011)), facilitating the location of antecedents; and because the analysis by *Tonalities* helps to illuminate several of the issues considered here. I take as my source Example 6a from (Russ, 2004), an article which examines a number of "analytical examples from the *Tonalities* project", some of which (including his Example 6a) manifest HEX, OCT and WT organization (2004, pp. 216–17).

I shall discuss both the musemic organization of the passage and the resultant tonal structure. In considering the former, the antecedents from which the musemes (fourteen are selected) manifesting HEX, OCT and WT organization derive are posited as *possible* not *definite* sources; it is not always possible to link musemes definitively in a nexus of imitation, such that a copy in a later work can be said unequivocally to derive from a given antecedent source composition.⁷ The antecedents are chosen because (i) they represent chronologically anterior works which Mahler may well have known directly; or (ii) because the musemes they contain may have been known to Mahler via one or more intermediate works; or (iii) because the musemes they contain constitute quite generic patterns which one might reasonably presume to have been widely propagated at the time.

4.2. Analysis

Example 1 i d shows a piano reduction of Mahler's passage. Above this are placed three further staves, i a containing the harmonic reduction suggested by *Tonalities*, i b showing HEX connective gamuts, and i c showing OCT connective gamuts.⁸ In the latter two staves, beamed pitches, some with brackets, are those reported by *Tonalities* while bracketed noteheads are my additions to make up the complete HEX or OCT collection. Example 1 i d also indicates fourteen of the constituent musemes, labelled M1–M14, using the symbology from (Jan, 2007, pp. 49–52). Those museme pitches mapping onto the associated connective gamuts are enclosed by a triangle in the case of HEX connectives and a square in the case of OCT connectives.

⁶ A similarly conceived analysis, of part of Debussy's "Nuages" from *Nocturnes* (1897–1899), is given in (Jan, 2014, sec. 2.3).

⁷ To identify the fourteen musemes, I looked for patterns which not only stood out visually in Mahler's score but which were salient aurally (in performance and imagination) and which therefore had, partly on account of their clear initial and terminal nodes, strong gestalt-psychological identity. I then located coindexes by searching for other instantiations of the posited musemes' scale-degree sequences via the *Themefinder* web resource (Huron, Kornstädt, & Sapp, 2015) and selecting appropriate matches.

⁸ Because *Tonalities* calculates connective gamuts from trichordal subsets of harmonies in adjacent segments, the scally-arranged gamut here may align the "wrong" pitch with the music below.

Below Example 1 i d, in boxes, is shown *Tonalities*' interpretation of the set-class of each segment. Note that, unlike the harmonic reduction, which makes a distinction between chord and non-chord notes, this set-class calculation is based upon all the pitches within the segment.⁹ Finally, Example 1 i e shows a quasi-Schenkerian voice-leading reduction which attempts, despite the intense chromaticism of the language, to differentiate between structural and prolongational elements. Bars 18–22 of this extract are the most complex, and it is upon these I shall focus. In order to distinguish it clearly, analytical output from *Tonalities* will be presented in Courier font.

Example 1: Mahler, Symphony no. 10 in F# (1910), I, bb. 16–24

i) Analytical Particella:

- a) Harmonic Reduction
- b) HEX Connective Gamuts
- c) OCT Connective Gamuts
- d) Piano Reduction and Musemic Analysis
- e) Voice-Leading Reduction

ii) Antecedent Coindexes of Musemes 1–14:

- a) Antecedent Coindex of M1: Johann Strauss II, *Der Zigeunerbaron* (1885), Overture, bb. 180–6.
- b) Antecedent Coindex of MPx (M2 + M3): William Monk, “Abide with Me” (1861), bb. 10–12.
- c) Antecedent Coindex of M4: Haydn, String Quartet in B \flat major op. 76 no. 4 (c. 1797), IV, bb. 34–6.
- d) Antecedent Coindex of M5: Brahms, String Quartet in A minor op. 51 no. 2 (c. 1873), IV, bb. 0–4.
- e) Antecedent Coindex of M6: Chopin, Mazurka in A \flat major op. 50 no. 2 (1842), bb. 8–12.
- f) Antecedent Coindex of M7: Schumann, *Kinderszenen* op. 15 (1838), “Kuriose Geschichte,” bb. 0–2.

⁹ Russ makes a distinction between “chord-like” and “note-class[-]set-like” segments (2004, p. 217). While it is the latter type which justify the use of pitch-class set-theoretical analysis, most segments in the passage under consideration are best regarded as examples of the former.

- g) Antecedent Coindex of M8: Suppé, *Dichter und Bauer* (1846), Overture, bb. 1– 4.
- h) Antecedent Coindex of M9: Lyadov, *Kikimora* op. 63 (1909), bb. 151–6.
- i) Antecedent Coindex of M10 and M13: Beethoven, Symphony no. 1 in C major op. 21 (1800), I, bb. 13–20.
- j) Antecedent Coindex of M11: Brahms, Violin Sonata in G major op. 78 (1879), I, bb. 1–4.
- k) Antecedent Coindex of M12: J.S. Bach, Fugue in C# minor from *Das wohltemperirte Clavier* Book I (1722), bb. 1–6.
- l) Antecedent Coindex of M14: Johann Strauss II, *Wiener-Bonbons* op. 307 (1866), no. 3, bb. 177–84.

M1 (b. 18) – a scale-degree sequence $\hat{3}\downarrow\hat{7}\uparrow\hat{2}$ – might derive from such patterns as are shown in Example 1 ii a. As stressed above, I am not asserting that the suave tune from Strauss’s Overture (appearing later in “So voll Fröhlichkeit gibt es weit und breit”) is the direct and unmediated source of Mahler’s line; there may well be intermediate stages of replication, or it may even be completely unconnected to it. Nevertheless, there seems no reason to reject the proposed connection outright, given Mahler’s undoubted familiarity with the music of Johann Strauss II (and his often ironic use of “Viennese” idioms); and especially when one considers the close general similarity of contour (embracing no fewer than eleven notes), indicated by the long bracket above the staves of the two passages. While little concrete trace of “So voll Fröhlichkeit” remains in Mahler, it seems nevertheless to guide the passage’s musemic sequence.

M1 contains the three pitches A#, E# and G#, the first two of which belong to the major seventh on F# [F# A# C# E#] chord of \$3, and the third to the dominant minor ninth with suspended fourth on D# [D# E G# A# C# / A B D] which *Tonalities* reads at \$4.¹⁰ *Tonalities* attempts to select a connective gamut to link these two harmonies. Finding the F# major collection [F# A# C#] read as connective across \$1–\$2 and \$2–\$3 invalid, it decides that the octatonic collection 0 [C# D# F# (Fx) A#] (the subtype known as “OCT_{0,1}”) is the best option.¹¹ A human observer, by contrast, might invoke an F# pentatonic collection as a more appropriate – certainly a more conservative – theoretical vantage point.

¹⁰ Within a square bracket, pitches after the forward-slash are interpreted as non-chord elements.

¹¹ Noteheads in brackets in Example 1 i b and c which are not linked by a stem to the beam are my additions, supplying the remaining pitches of the particular collection. Bracketed noteheads which are linked by a stem to the beam are those which are bracketed in *Tonalities*’ output, such as the F* in “[C# D# F# (Fx) A#].”

Below M1 in Example 1 i, two further musemes, M2 and M3,¹² might be read in the alto and bass lines of the texture, associating to form a complex labelled Musemplex (hereafter “MP”) *x* (as indicated by the vertical brackets linking the horizontal beams of the musemplex’s components). While these musemes may well occur independently in other contexts (as symbolized by the “//” divider superimposed on the vertical brackets¹³), their existence as a musemplex is supported by their co-replication in other contexts. As a common figure in nineteenth-century tonality, their appearance in “Abide with Me” (Example 1 ii b), while possibly not known to Mahler (but he may have heard it during his visits to New York, the first of which was in 1907), is nevertheless suggestive of their general prevalence. The hymn version cadences in its local ii (F minor); Mahler’s version mutates the third vertical of the musemplex (\$5) to a minor added #4 collection on B [B D F F# / G#] within a B melodic minor scale [B D F# G# / F] prolonging gamut.

On this reading, the $g\sharp^1$ is an appoggiatura to the following chord note, $f\sharp^1$; but in musemic-evolutionary terms the vertical on the first quaver of b. 19 might be read as relating historically to the tonicized ii of the antecedent (as an inversion, ii^6_3 , of the ii^5_3 form in the hymn) in F#, the $g\sharp^1$ therefore being a chord note. Such is the degree of chromatic mutation in the Mahler passage, that several multiple readings are possible; these are regulated by various verbal-conceptual memplexes (some operationalized, in ways which are still not fully understood, in *Tonalities*’ Visual Basic code) which, in a memetic view, would defer to a reading based on evolutionary-homological criteria.

The first two elements of MP*x* also contain pitches drawn from OCT_{0,1}, except for the b^1 at b. 18³. This functions as an appoggiatura to the chordal $a\sharp^1$ at b. 18⁴ and illustrates the dichotomy, even tension, between *Tonalities*’ predominantly reductive, pitch-enumerative approach and the linear-historical perspective provided by tracing musemes as they (sometimes) overlap segment boundaries. Apropos the hypotheses advanced in Section 3.2, the conglomeration of M1, M2 and M3 here illustrates IMVJ. Note that M1 does not illustrate IMM: it is DIA in F# major, but is also susceptible to an OCT reading by virtue of its DIA-disrupting association with MP*x*.

M4, despite the chromatic material above and below it, retains a connection with the tonic, outlining an F# (minor) shape which opens with the $\hat{2}\downarrow\hat{1}$ appoggiatura referred to in connection with M2 – the two musemes intersect on this pitch, $g\sharp^1$ (Jan, 2007, pp. 74–7) – and which closes with a $\hat{1}\uparrow\hat{6}\downarrow\hat{5}$ figure perhaps ultimately originating in Baroque fugue subjects. A possible antecedent coindex, from Haydn, is shown in Example 1 ii c, although the initial node of the earlier form is not particularly salient. The sequential juxtaposition/elision of MP*x* with M4 across the barline into b. 19 prompts *Tonalities* to invoke a connective hexatonic collection 3 [D# F# (G) A# B D] (HEX_{2,3}) to link the thirteenth chord of \$4 with the minor added #4 collection of \$5. The $g\sharp^1$ which is the intersection point between M2 and M4 is, as

¹² M3 is primarily understood as a melodic museme here, but, as a bass-line, it clearly carries additional harmonic implications, reflected in the continuity of the note stems at this point in Example 1 i d to the top of the note-stack.

¹³ Logically, the absence of such a bracket would indicate the presence of a multi-voice museme, not a musemplex composed of a number of single-voice musemes.

noted, not a chord note in *Tonalities*' interpretation, and therefore not a member of HEX_{2,3}. A desire to include the g^{#1} might prompt a human interpreter alternatively to invoke an enharmonic D[#] harmonic minor collection as the connective, which also would afford a more normative vantage point than HEX_{2,3}.

M5 is problematic because one interpretation of its scale-degree sequence is $\hat{3}\downarrow\hat{2}\downarrow\hat{7}\downarrow\hat{2}$ in C minor, despite the clearly non-C-minor harmonic context. Alternatively, it could be read as an enharmonically renoted $\#3\downarrow\#3\downarrow\hat{1}\downarrow\hat{3}$ in B minor, which offers a more logical connection to the harmony of \$5. The coindex shown in Example 1 ii d – which, however, accords with the former scale-degree interpretation – differs in its terminal pitch, but it might be possible to read this as a substitution of $\hat{5}$ for the reiterated/lower-octave $\hat{2}$. In their sequential juxtaposition, M4 and M5 illustrate IMHJ. The shift from the minor added #4 collection of \$5 to the dominant minor thirteenth on D[#] [D[#] E Fx B C[#] / Eb E[#] D] of \$6 is again impossible to rationalize in DIA terms, and is seen by *Tonalities* as organized by another HEX connective, again hexatonic collection 3 [D[#] F[#] Fx (A[#]) B D] (HEX_{2,3}). Note that we are dealing with a chord progression from \$5–\$6 which, while it may well be musemic,¹⁴ is not labelled as such here (unlike M7, M10, and M13, considered below); but these chords are to some extent expressed by the musemes sounding above them, albeit to a lesser extent that would normally be the case in earlier styles.

M6 is a mutation of such passages as are shown in Example 1 ii e, the antecedent lacking only the $\#5$ found in the Mahler passage. M5 and M6, being associated with the progression from the dominant minor thirteenth on D[#] [D[#] E Fx B C[#] / Eb E[#] D] of \$6 to the major triad on A [A C[#] E / D[#] F[#]] of \$7, also illustrate IMHJ – and indeed IMVJ if their elision is taken into account. The normative resolution of the V¹³ is to a G[#] minor chord, and so the major triad on A, as \flat II in G[#] minor, is strikingly non-DIA in this local context. Segments 6 and 7 are bridged by a connective octatonic collection 0 [C[#] D[#] E Fx A (A[#])] (OCT_{0,1}). The mutation of M6 vis-à-vis its posited antecedent (adding the $\#5$ of the A major chord forthcoming in \$7), while strictly an example of IMM, nevertheless does not tie it much more closely to the OCT connective which governs this transition.

M7 is the common tonal progression from I to V⁴₃, with attendant 10–10 linear intervallic pattern between $\hat{1}/\hat{3}$ and $\hat{2}/\hat{4}$, although here $\hat{3}$ and $\hat{4}$ are in an inner voice. A possible antecedent is shown in Example 1 ii f, with $\hat{1}/\hat{3}$ and $\hat{2}/\hat{4}$ in the outer voices. M6 and M7 may constitute a musemplex, but they are not linked by brackets in Example 1 i d because I have not located a coindex in which they are similarly associated. M7 is DIA in A major. Its two vertical components, in association with the main part of M6 are linked by a connective E major collection [E G[#] A B C[#]], giving the DIA by IMVJ which is normative and unremarkable in tonal music. This reading is perhaps partly motivated by the chromatic passing tone d^{#1} (b. 20²) of M6 (its e^{#1} in b. 19⁴ comes into the same category, but is obviously not invoked in this particular connective).

¹⁴ Indeed, the chord progression and the associated musemes might form a musemplex, the verification of which, for the sake of expository and visual clarity, is not pursued here.

M8 outlines scale degrees $\hat{7}-\downarrow\hat{6}-\downarrow\hat{4}-\downarrow\hat{6}$ in the local V of A major context of \$8. Its possible antecedent, shown in Example 1 ii g, articulates the same harmony using the same scale degrees. However, Mahler's replication displaces the final $\hat{6}$ of Suppé's museme ($\hat{7}-\downarrow\hat{6}-\downarrow\hat{4}-\uparrow\hat{6}$) by an octave. It appears the case that such octave displacements are common mutational strategies in Mahler and his younger contemporaries, lending normative antecedents a more angular, tortured character in their mutated form (see also, for instance, the opening melody of Berg's song "Die Nachtigall" (from *Sieben frühe Lieder* (1905–1908))).

M9 might be read as the scale-degree succession $\hat{3}\uparrow\hat{5}\downarrow\hat{3}\downarrow\hat{5}$ in Mahler's local B_b major tonal context of \$9 ($\hat{2}\hat{6}\uparrow\hat{1}\downarrow\hat{2}\hat{6}\downarrow\hat{1}$ in the now-distant tonic F \sharp). A possible antecedent coindex is given in Example 1 ii h, from Lyadov's *Kikimora*, where it outlines scale-degrees $\hat{1}\uparrow\hat{3}\downarrow\hat{1}\downarrow\hat{3}$ in a clear A minor context.¹⁵ In the same fashion as the sequences formed by M4–M5 and M5–M6, M7/8–M9(10) illustrate IMHJ. The tritonally-related dominant major ninth on E [E F \sharp G \sharp B D / C \sharp D \sharp] of \$8 and the major triad on B \flat [B \flat D F / G A] of \$9 are, perhaps predictably, linked by a connective octatonic collection 1 [D E F G \sharp B \flat B] (OCT_{1,2}). From the vantage point of older theory, the harmonies of \$7, \$8 and \$9 might alternatively be understood as a somewhat unsyntactic I–V⁴_{3–b}–II⁵₃ in A major or, considering \$8 and \$9 only, as \flat II⁴₃–V in E \flat minor. In these readings, a local tonal-field perspective would override the connective-gamut-based model employed by *Tonalities*.

The two-chord progressions of M10 (b. 21) and M13 (b. 22) are broadly analogous, despite the difference between the chords in \$9 (major triad on B \flat) and \$11 (dominant major ninth collection on D \flat [D \flat E \flat F A \flat B / D]), owing to their root motion by falling minor thirds and chromatic inner-voice motions $b\flat$ (implied)– $b\sharp$ and $d\flat$ ¹– $d\sharp$ ¹ respectively, which give rise to false relations across the middle of the bars. As their antecedent in Example 1 ii i indicates, they may derive (the former more clearly than the latter) from secondary-dominant-type tonicizations of ii in progressions such as I–V/ii–ii–V–I.

M11 (\$10) might be said to derive from such patterns as are shown in Example 1 ii j – a falling arpeggio $\hat{3}\downarrow\hat{1}\downarrow\hat{5}$ preceded by an appoggiatura $\hat{4}$.¹⁶ Again illustrating IMHJ in relation to M9, M11 falls within a dominant minor thirteenth on G [G B E \flat F / A C \sharp] (\$10). It is connected to the previous major triad on B \flat (\$9) by an octatonic collection 1 [D F G B \flat B] (OCT_{1,2}). Clearly M10 encompasses this collection – hitherto, musemes have not aligned so closely with connectives – and is therefore the most strongly OCT of the musemes considered so far. The alteration to which M11 is subjected in the Mahler passage (vis-à-vis its proposed antecedent) turns the normative DIA collection [1/ $\hat{5}$, (3), (5), 6/ $\hat{1}$, (8), 10/ $\hat{3}$, 11/ $\hat{4}$] into the WT₁ [1, (3), 5, (7), 9, 11], an example of IMM. Note that,

¹⁵ In the case of such style form/shape-type musemes, it may well be the case that composers alighted upon them independently, as evolutionary "good tricks" (Dennett, 1995, pp. 77–8), rather than imitating them from a specific antecedent coindex.

¹⁶ It should be noted that M11 has an affinity of contour with M5 and M8 and later with M14 – indeed the music before the passage in Example 1 i contains similar figures in the second halves of the bar – and so there are inevitably intraopus (idiostructural, developmental) processes at work here as well as the extraopus (memetic) ones with which I am primarily concerned.

Table 1: *Tonalities'* “Summary of Analysis” for Example 1 i d

These data afford robust evidence for the basic thesis here. The juxtaposition and mutation of musemes creates a challenge to a DIA-orientated theory, and only the intercession of new theoretical paradigms – in this case *Tonalities'* deployment of OCT and HEX connectives – facilitates rationalization of the tonal organization. The sudden harmonic lurches across the barline and across the middle of the bar typical of this passage demarcate a series of localized (2-beat-long) DIA “islands”, whose connection to form a larger “archipelago” often relies on the invocation of OCT and HEX collections in order to bridge the turbulent waters between them. As the third column of Table 1 makes clear, OCT_{0,1}, OCT_{1,2} and HEX_{2,3} are important as connective gamuts, taking second and third place after F# major. OCT_{1,2} is particularly important, serving as a connective in over a quarter of segment-to-segment transitions. This phenomenon appears much less common in most music of the period 1700–1875 and *it is arguably largely driven by musemic juxtaposition and mutation*. Thus, while extant theory has argued eloquently for the necessity of expanding the frame of reference to incorporate non-DIA collections in order to understand much of the music of the period 1875–1925, what I believe is evident from the discussion here is that such theory misses a crucial point: this expansion *must be yoked to a Universal-Darwinian framework* (Dawkins, 1983; Plotkin, 2010) in order fully to understand the forces driving the aetiology and evolution of both low-level and system-level change in music.

As might be expected, the voice-leading graph in Example 1 i e struggles to impose a linear-DIA unity on the passage's relentless chromaticism. It is largely guided by *Tonalities'* assessment of the prolonged harmonies, but their constituent pitches do not always give the smoothest analysis from a linear perspective. Therefore, pitches which *Tonalities* regards as non-chordal are occasionally included on the voice-leading graph because they make greater contrapuntal (and memetic) sense. An example of this is the $d\sharp^2$ of b. 19⁴, the inclusion of which is justified by the following A major harmony, and in relation to which it might be heard as a $\hat{4}$. The graph illustrates the point made in Section 3.2 that HEX and OCT phenomena tend initially to be situated at the foreground and shallow-middleground levels and that deeper structural levels – certainly in this particular passage – tend to remain essentially DIA.

Thus, it is important to note that, just because *Tonalities* often reads OCT (less often HEX) *connective* gamuts in this passage, the music is not OCT in the way that certain passages of, for example, Stravinsky or Schoenberg composed a few years after this symphony are. For this to be the case, an OCT (or HEX) collection would have to be interpreted as a *prolonging* gamut within segments – but this does not occur in any of the sixteen segments shown in Example 1 i d. Instead, *Tonalities* prefers to read various DIA prolonging gamuts within segments, such as the G# diatonic minor scale [G# A# B C# D# E / A D] of \$4. Nevertheless, historically speaking, the direction of travel is quite clear, and so if later music were to be considered in the light of the analytical methodology utilized here, it would be likely to demonstrate increasing use of non-DIA prolonging gamuts and, therefore, deeper structural chromaticism.

5. Conclusion: Problems and Prospects for the Memetic Analysis of Tonal System Evolution

While this article has merely scratched the surface of a highly complex issue, I hope to have shown – albeit necessarily by a suggestive example rather than by overwhelming weight of evidence – that tonal systems are complex entities deriving their properties from the attributes of their smallest constituent elements. Juxtapositional and mutational changes to these elements inevitably have knock-on effects on the larger system which, in the case of musemes, pose a challenge to existing music-theoretical models. The latter must themselves evolve in order to be seen to function as an effective model for the new systemic settlement. As noted, I have downplayed this second element in order to concentrate on specifically musical processes; but a comprehensive treatment of the issue would need to concentrate on the development of theoretical perspectives which acknowledge HEX, OCT and WT as viable systems of organization and which model them effectively.

A more fundamental omission here is the deeper problem of why certain musemes coalesce in particular ways in order to generate specific harmonic progressions. The answer can, in Blackmore's opinion, be found only by taking what she terms the "meme's eye view" (Blackmore, 1999, p. 37). That is, instead of asking why a composer chose to arrange (IMHJ, IMVJ) or change (IMM) a series of musemes in a particular way, the museme's eye view requires us to ask *what attributes of the musemes themselves* prompted the composer to recall, rework and reemploy them in a particular context. Answering this question not only requires one to address the issue of the perceptual-cognitive salience of musemes, but also to consider the nature of the initial and terminal nodes of the musemes concerned and the variety of juxtapositions, within and without musemeplexes, such nodes permit. A preliminary treatment of these ideas is offered in (Jan, 2010).

Further development of the ideas presented here would also require fuller consideration of structural-hierarchic and voice-leading factors. In particular, the issue of how IMHJ, IMVJ and IMM affect musemic structure at middleground and background levels needs further theorization. While *Tonalities* is to some extent sensitive to voice-leading patterns, further refinement of its processing algorithms might usefully endow it with the capacity to assign hierarchic value to pitches and therefore to develop the capacity to distinguish more subtly between structure and prolongation, even though, paradoxically, the music for which the software is perhaps most illuminating is increasingly difficult to conceive in terms of this distinction (Pople, 2004, pp. 162–4).

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