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Jan, Steven

Similarity Continua and Criteria in Memetic Theory and Analysis

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1. Introduction: On Similarity

Similarity is one of the most important aspects of the production and reception of music. If a musical work manifests too much internal similarity (in the form of various types of repetition), it may be regarded by listeners as uninventive. If it manifests too little internal similarity, it may be thought incoherent. When the degree of similarity between works (by the same composer or by different composers) is considered, the focus moves to the diversity of patterning encompassed by the resulting style, at whatever hierarchic level (Meyer 1996: 23–4). Composers have naturally considered the degree of similarity within and between their works, and between their works and those of others, very carefully and, as a result, the issue has been of considerable and enduring interest to those who study music from various scholarly perspectives. To the extent that these interconnected research programs can be separated, they include the musicological (Reynolds 2003), the psychological (Wiggins 2007; Ockelford 2009), the evolutionary-psychological (Minsky 1981), the music-theoretical/analytical (Cone 1987), and the computational (Crawford et al. 1998). Similarity even has a legal dimension in some contexts, as in the case of copyright infringement lawsuits, usually involving popular music (Cronin 1998; Müllensiefen and Perdzich 2009).

To focus upon the three perspectives of greatest relevance to this article, from a psychological perspective, similarity within a movement or work is important in that it allows listeners to build a mental representation during hearing (Zbikowski 2002; Ockelford 2013), perhaps abstracting ‘cues’ to serve as helpful markers of the work’s sequential and hierarchic structure (Deliege 2000; Jan 2010). Without such assistance, listeners might, as suggested, find the music ‘cognitively opaque’ (Lerdahl 1992: 115). From a music-theoretical/analytical perspective, similarity is a marker of recurrent musical techniques, devices, and structural plans, be this recurrence within a movement or work (the province of analysis) or between works (the province of theory), the latter shading into music-historical/stylistic trends and developments. From a computational perspective, similarity detection algorithms have various objectives, one of which is to model human perception of similarity as part of a wider enterprise aimed at simulating various aspects of music cognition (Temperley 2001). Other uses
are more musicological, including attribution, source-location, and tracing relationships between compositions (Wiering et al. 2004: 116). From this last standpoint, resolution of the issue of similarity is central in questions of musical influence, borrowing and intertextuality (Korsyn 1991; Burkholder 1994; Klein 2005). Research in both the psychological and the musicological uses of computerised similarity detection is often conducted under the rubric of music information retrieval (MIR) (see Velardo et al. n.d.) for an overview of recent work, progress in this field being stimulated by the annual MIREX competition (MIREX 2014), of which symbolic (in addition to audio-based) melodic similarity is a component.

Similarity has particular importance in the application of memetics to music (for a detailed account, see Jan 2007). Memetics is a theory of cultural evolution which regards a discrete unit of cultural information (a 'meme') as, in many ways, analogous to the gene (Dawkins 1989; Blackmore 1999). Both replicators, once they come into existence, are subject to the 'evolutionary algorithm' of variation, replication, and selection (Dennett 1995: 343), and they are capable, progressively and incrementally, of building 'systems of great complexity' in their respective realms (Dawkins 1989: 322). Memetics serves as the background to this article and, it is asserted, offers a means of uniting the psychological, the theoretical/analytical, and the computational perspectives on musical similarity mentioned above under a unified framework. But what might it offer – generally to musical scholarship and to the specific issues considered here – beyond existing music-theoretic approaches? While an extended rationalisation is outside the scope of this article (for this, see Jan 2007: 1–4; 2010: 5–8), a brief summary, in the form of the following sequential list, might be found useful.

(i) Memetics accords with the 'Universal Darwinism' (Dawkins 1983b; Plotkin 2010) which some believe regulates and relates pattern replication in the physical, biological, and cultural realms. As a metatheory of culture, it is validated by its alignment with one of the most powerful shaping forces on earth (and presumably beyond), namely Darwinian selection.

(ii) It proceeds from the simple principle that our gene-controlled psychological attributes determine which musical patterns we are or are not able to perceive, cognise, and remember – and therefore which musical patterns can or cannot exist.

(iii) It recognises that, given a crowded culture and a finite supply of memory, the evolutionary algorithm will inevitably begin to operate upon these patterns, which consequently become aetiologically, epistemologically, and ontologically privileged.

(iv) Indeed, it argues that this Darwinian view of musical patterning trumps extant music-theoretic conceptions of what constitutes a unit in music. The latter often invoke higher-order, more abstract categories (some of which are themselves memes). In this sense, replication reifies a pattern and affords it a certain privileged status.

(v) This way of seeing the musical world opens up new perspectives on musical structure and style. To paraphrase Dobzhansky, 'nothing in music makes sense except in the light of memetic evolution' (Dobzhansky 1973).

In other areas where memetics is applied to culture, such as in the social sciences, similarity is often less stringently tested, perhaps because it is less easily quantified. Two ideas or two artefacts, for example, might be held to be similar only in a loose way, because it is difficult to quantify how closely corresponding two differently worded sentiments or two differently constructed vases really are. In music, by contrast, similarity in certain aspects can be assessed fairly strictly and accurately. Such judgements are important in memetics for resolving the distinction between the evolutionary concepts of analogy and homology (Dennett 1995: 357). That is, they help to determine whether a resemblance between two patterns is accidental/fortuitous (which may, of course, still be interesting), or is the result of memetic transmission, respectively. In many situations, the greater the similarity between two patterns, the greater the likelihood that their relationship is homological as opposed to analogical.

Beyond developing and refining a memetic approach to understanding music in general, the aim of this article is to reframe composer and music theorist David Cope's categories of similarity relationship, and also the 'Earth Mover's Distance' metric of Rainer Typke and others, in explicitly memetic terms, in order to help model and quantify similarity in evolutionary terms and thereby to help distinguish between analogy and homology in similarity relationships. Achieving this aim will not, of course, resolve all of the outstanding issues in the application of memetics to music, but it does offer a way into addressing some of the most interesting and pressing ones. To this end, Section 2 looks at various ways of classifying similarity, focusing on Cope's notion of different types of affinity, or 'allusion', between works. Section 3 explores Cope's categories in more detail, relating them to the precepts and predictions of memetics. Section 4 discusses the quantification of similarity in music, drawing upon the notion of 'transportation distance', and develops ways of using the Earth Mover's Distance metric for quantifying psychological, evolutionary, and neurobiological distances in memetics, the latter drawing
on William Calvin’s theories of information encoding in the brain. Section 5 examines a specific memetic case study, bars 111–27 of Beethoven’s Piano Sonata in B, major op. 106 *Hammerklavier*, in detail, attempting to apply ideas developed in the previous sections in order to measure specific instances of pattern mutation in the passage. Section 6 briefly considers some of the wider implications of the ideas discussed.

It might be argued that, after a wave of literature responding directly to Dawkins’ initial writings on the meme (Dawkins 1983a, 1989; Dennett 1995; Lynch 1996; Blackmore 1999; Aunger 2000, 2002), recent years have seen a falling off of interest in memetics. Some might see this as manifested most directly by the demise, in 2005, of the *Journal of Memetics*. While this is difficult to prove (for a methodology, see Jan 2014), if indeed true it might be the result of a feeling that memetics has not provided accounts of culture which are sufficiently robust and illuminating to persuade scholars in a number of relevant disciplines (including music theory and analysis) that it is a metatheory worth adopting. To use a political metaphor, memetics has not (yet) delivered on its often extravagant promises. As a second critique, some would also bemoan the lack of evidence for the physical existence of memes, wanting to know what a meme – in its ‘memotypic’ as opposed to its ‘phemotypic’ form (Jan 2007: 28–31) – looks like.

Whether interest is abating or not, it appears that what is needed for memetics’ further development is a set of more specific theoretical and empirical studies, in order to provide greater solidity to its claims. These studies need to respond dialectically to each other, as particular theoretical refinements and extensions are advanced and then tested using real-world data. The present article is offered mainly in the spirit of the former (although, as noted at the end of Section 5.2.8, its claims lend themselves to empirical assessment); and one might cite work such as (Morin 2013) as contributing substantially to the latter. Ideally, such interplay between the *a priori* and the *a posteriori* in memetic research needs to be more tightly coordinated than has hitherto been the case, which is essentially an argument for even greater interdisciplinarity in future work in the field. As for the second critique, just as evolutionary theory made significant advances before an understanding of its natural-selective and then genetic mechanism was consolidated – and the gene is, arguably, still a fuzzy category (Dawkins 1983a: 81) – memetics can offer insights into musical structure and style change without knowledge of the precise nature of the meme. Nevertheless, it might be argued that the issues discussed in Section 4.4 go some way to addressing this concern directly.

2. Similarity Continua in Memetics

Starting with a quite generic definition, one might say that similarity in music occurs (and is perceived) when there is a (cross-)mapping of the attributes of two patterns in one or more musical parameters which exceed some arbitrary threshold. Mappings below this threshold mean the patterns are regarded as dissimilar. Naturally, the parameters selected for comparison will have a bearing on the determination of similarity, as will the criteria imposed for judging the existence or the extent of mapping. In music, similarity may be manifest across a range of parameters, but pitch and rhythm, as primary parameters, lend themselves most readily to such determinations, as compared with the secondary parameters of texture, timbre, and dynamics (Meyer 1992: 476; 1996: 14). This is because what might be called ‘digital’ comparisons (‘is the note in position n of these two patterns an x?’) are normally easier to make than ‘analogue’ ones (‘is the texture in position n of these two patterns of equivalent thickness?’) (Selfridge-Field 2004: 94). In some ways this mirrors, within music, the distinction between music and other realms noted in Section 1.

Certain attributes of *musemes* may be plotted against the number of their component elements. The latter are understood to mean discrete note events, generalised as pitch-rhythm complexes. *Figure 1* (after Jan 2007: 62, Figure 3.2; 2011: Sec. 4.1.1) summarises these relationships, taking its X-axis (like the Y-, on an Aristotelian low-median-high scale) to measure a number of related, but not directly comparable, conceptual categories.

*Figure 1* is based on the premise that there is a continuum of museme-length, measured in terms of contiguous notes. At the short (c. three-element) end of this continuum are units of low salience but high replicative stability over time, and, at the long (c. nine-element) end, are units of high salience but low replicative stability over time. The more elements a museme consists of, the greater its salience, memorability, and individuality. This is not indefinitely true, for a point is reached after which the constraints of human memory mean that a diminishing-returns effect comes into play, and a pattern becomes difficult to parse and store. A ‘Millerian optimum’ is likely, conforming to the ‘magical number seven, plus or minus two’ asserted to be the effective limit of short-term memory (Miller 1956; Snyder 2000: 36).
Figure 1. Attributes of melodic musemes plotted against number of component elements

<table>
<thead>
<tr>
<th>Number of Museme Elements</th>
<th>High</th>
<th>Median</th>
<th>Low</th>
</tr>
</thead>
</table>

Potential copying-inbility

Perceptual-cognitive salience

LT Memorability/ST Non-Memorability

Perceived individuality (style structure) vs. generality (style shape)
(Narmour, 1990, p. 34)

Information content (given by r value) (Jan, 2007, p. 61)

Perception as quotation/borrowing/plagiarism

Likelihood of origin by homology/non-analogy;
Unlikelihood of origin by analogy/non-homology

[Cope’s listener recognition/referential-analytic continuum:
Perceived as quotation rather than commonality
Perceived as commonality rather than quotation
\[\leftrightarrow\] framework \[\rightarrow\]
\[\leftrightarrow\] likeness \[\rightarrow\]
\[\leftrightarrow\] paraphrase \[\rightarrow\]
stylistic non-integration (Cope, 2003, p. 11)]

Strictly, this refers to the number of perceptual categories that can be typically distinguished (see also Baddeley and Hitch 1974; Baddeley 2007). Additionally, a ‘musemic optimum’, implementing Miller’s ‘minus two’, is hypothesised, on account of certain cultural constraints operating against direct quotation. Temperley’s ‘PSPR 2 (Phrase Length Rule)’ – which says listeners generally ‘prefer phrases to have roughly 8 notes’ – hypothesises a figure which appears slightly too high in Miller’s terms (Temperley 2001: 69).

The bottom part of Figure 1 refers to Cope’s categorisation of similarity according to various attributes. It is perhaps unsurprising that Cope has investigated this question so thoroughly, given that the principal focus of his various research projects – the computer synthesis of musical style arising from his Experiments in Musical Intelligence (EMI) (Cope 1996; 2001) – relies on understanding the nature of the patterning which is to be replicated in the process of synthesis. It is therefore worth remembering that Cope’s overriding motivation...
is to understand, by synthesis, the nature of the compositional process, in part to inform his own compositional praxis. His supporting analytical research is conducted on the basis that, in Meyer’s definition, ‘...style is a replication of patterning, whether in human behavior or in the artifacts produced by human behavior, that results from a series of choices made within some set of constraints’ (Meyer 1996: 3). Table 1 shows Cope’s various categories of ‘allusion’, these being identified by a process he terms ‘referential analysis’, implemented by a computer system named Sorcerer (Cope 2003: 27; 2014); and it relates them to the hypothesised attributes of categories of musemes posited in (Jan 2011; see also Jan [n.d.]). Certain concepts in Table 1 are explained in later sections.

Whereas Cope organises his categories on a continuum (Commonalities … Frameworks … Likenesses … Paraphrases … Quotations), it might be argued that the three middle categories are not length-specific. That is, whereas a Commonality needs to be short for it to fit smoothly into the numerous contexts in which musical ‘connective tissue’ is needed, and whereas a Quotation needs to be of a certain length for its status as such to be recognised by the listener, a Framework, a Likeness, or a Paraphrase may occupy either end of this scale, although many seem to occupy some middle range.

It might be argued that the longer a duplicated passage the less likely it is to be memetic, on the grounds that a Quotation does not constitute genuine replication. From a strictly Darwinian standpoint – from the ‘meme’s eye view’ (Blackmore 2000) – whether the passage replicated is a Commonality or a Quotation is immaterial. What matters from this perspective is the survival by replication of a museme. Whether it exists as an unreflectively incorporated generic figure or a distinctive pattern consciously extracted by a composer from a work of his/her immediate cultural context, for whatever motive, is irrelevant from a meme’s metaphorically selfish perspective (for a discussion of gene selfishness, see Dawkins 1989).

3. Cope’s Categories of Allusion Reimagined in the Light of Memetics

Cope’s categories are not always as clear-cut as he presents them, and sometimes a resemblance may involve aspects of more than one category, giving rise to intersection between categories (Section 3.6). It is his middle three categories – Paraphrase, Likeness, and Framework – which are most difficult to resolve, owing to their tendency to encompass phenomena which overlap with each other in various ways. Indeed, all five categories are not necessarily objectively distinguishable, and different listeners might place a given correspondence in different categories. A second complication is that a given pair of corresponding musical segments might map in a number of different ways against each other, encompassing different categories of correspondence perhaps on account of their consisting of a number of musemes. Therefore, a distinction needs to be made between potentially multivalent resemblances connecting two or more phrases (which may each consist of a collection of musemes) and resemblances between individual musemes across two or more phrases. This issue is pertinent in all but the last of Cope’s categories, Commonality. As a third complication, not all allusions are necessarily gestalt-partitioned musemes: an ‘overlapping’ portion may straddle a museme segmentation boundary (Jan 2007: 74–7); or – in the case of ‘coindexation-determined segmentation’ (Jan 2011: Sec. 4.1.2) – may function as a museme despite such straddling (Bod 2001). A fourth, but probably not final, complication is that, because memetic transmission is rarely completely accurate, mutation means that perfect quotation (in its broadest sense) is unusual for all but the most direct of Quotations (in Cope’s sense) and the most simple of Commonalities. As discussed in Section 3.5, there are also questions about whether such Commonalities are actually transmitted memetically.

All of this suggests that a rather broader set of relationships obtains for inter-work (memetic) connections than those which govern the intra-work (motivic) recurrences important for cognition of the structural coherence and narrative unfolding of movements. For instance, a museme in two separate works need not necessarily adhere to Temperley’s stipulation ‘that, for two [intra-work] segments to be recognised as motivically related, they must be (1) related in pitch pattern …, (2) identical in rhythm …, and (3) metrically parallel’ (Temperley 2001: 332–3).
<table>
<thead>
<tr>
<th>Cope’s Category</th>
<th>Definition</th>
<th>Museme Category</th>
<th>Definition</th>
<th>Stylistic Integration</th>
<th>Listener Recognition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quotations (Q)</td>
<td>‘often involve exact note and/or rhythm duplication’ ([Cope 2003: 11]).</td>
<td>Melodic, harmonic, and rhythmic musemes at the upper end of Miller’s continuum (U).</td>
<td>Melodic</td>
<td>Replicated localised, psychologically closed ([Snyder 2000]) &lt;7±2-element pitch sequences.</td>
<td>low</td>
</tr>
<tr>
<td>Paraphrases (P)</td>
<td>‘typically involve different pitches but similar intervals paired with rhythmic freedom’ ([Cope 2003: 12]).</td>
<td>Museme allele-classes and/or Musico-operational/procedural memes (RSE).</td>
<td></td>
<td>Replicated ways of operating upon musical materials.</td>
<td></td>
</tr>
<tr>
<td>Likenesses (L)</td>
<td>‘have different pitches, intervals, and rhythms but have some underlying similarity such as overall likeness of directions or interval sizes’ ([Cope 2003: 13]).</td>
<td>Museme allele-classes and/or Musico-operational/procedural memes (RSE).</td>
<td>See above.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frameworks (F)</td>
<td>‘involve the incorporation of interpolated notes so that potential similarity surfaces only after these notes are removed during analysis’ ([Cope 2003: 14]).</td>
<td>Formal/structural (Memesätze) musemes (U).</td>
<td>Repeatedly reinstanliated formal/structural archetypes and schemata, themselves consisting of harmonic/melodic patterning at deeper structural-hierarchic levels ([Jan 2010]).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commonalities (C)</td>
<td>‘typically involve patterns which, by virtue of their simplicity – scales, triad outlines, and so on – appear everywhere’ ([Cope 2003: 17]).</td>
<td>Melodic, rhythmic, and harmonic musemes at the lower end of Miller’s continuum (U).</td>
<td>See above.</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>low</td>
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</tr>
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</table>

| | Textural/timbral/dynamic (RSE). | Replicated combinations of instruments, registers, and timbres. |
| | Performative (RSE). | Replicated motor patterning in the service of improvisation or performance. |
3.1. Quotation

‘often involve exact note and/or rhythm duplication’. (See Table 1)

The implication of Cope’s term is that the unit replicated is of sufficient length to be understood in the mind of the perceiver and that of the individual replicating it as a distinctive entity taken from one readily identifiable context and transplanted to another such context. Quotations are perhaps most often melodic musemes, but the source-copy connection is often heightened by a ‘co-adapted’ (associated) harmonic museme. Cope’s implication, and that of Figure 1, is that a Quotation is a reasonably lengthy museme, on the upper end of Miller’s continuum, which is essentially replicated in its entirety in another context. While this is certainly true, it is also the case that short figures with high salience – this the result of some distinctive pitch, harmonic, or rhythmic ‘marker’ – may be perceived as Quotations. The listener might conclude, on the basis of the salience-affording element(s), that the pattern could not have originated from other than a specific source. With the rise of the work as an aesthetic object in the mid-eighteenth century (Dahlhaus 1982; Goehr 1992), Quotations, certainly those on the upper end of Miller’s scale, became increasingly uncommon, their aesthetic effect therefore heightened when they do occur.

Figure 2 represents Quotation schematically. In Figures 2–6, pitches are symbolised by black circles, and bars by vertical lines. The source museme is shown on the upper ‘stave’ and the copy on the lower. The museme pair are boxed in order to represent their demarcation from surrounding non-replicated pitches.

Cope’s Figure 1 is offered as an example of Quotation and hypothesises a relationship between b. 24 of Mozart’s Sonata K. 457, II and bb. 1–2 of Beethoven’s Sonata op. 13, II. He argues that ‘although only three melodic notes (C–B flat–E flat) exist in common between the two themes here, Beethoven’s use of the identical key (A-flat major) and nearly identical harmonization clearly reveal the Mozartean origins’ (Cope 2003: 12). Because a unit as short as this can only be regarded as a Quotation if it has sufficient markedness to create a strong association between source and copy forms, an anticipation of Beethoven’s distinctive V₄₃ harmony, b. 1² of his theme, in the Mozartean source might have been suggestive; but the harmonisation is not ‘nearly identical’, for Mozart uses a tonic pedal throughout (Jan [n.d.]).

Examples 1 (i) and 1 (ii) show a much more clear-cut instance of Quotation, with thirteen melodic pitches common to the two passages. The personal association between J.C. Bach and Mozart (the concerto was written shortly after Bach’s death), the shared tonality, and the nearly identical harmonisation and voicing make this homage unmistakable. It is probably best to regard the unit as a musemeplex rather than a single thirteen-note unit-museme. It is made up of distinct musemes which might be found replicated independently in other contexts – a suggested segmentation is indicated by boxes in the examples, but this oversimplifies the passage because several of the horizontal lines may exist as independent melodic musemes as well as constituting harmonic musemes – but the whole five-harmonic-
Example 1 (i). Quotation: J.C. Bach: Overture to *La calamita de’ cuori* T. 272/5 (1763), II, bb. 1–4

Example 1 (ii). Quotation: Mozart: Piano Concerto in A major K. 414 (386a; 385p) (1782), II, bb. 1–4

The museme complex is probably limited to these two instances. This is because its replicative opportunities in the works of late-eighteenth-century composers appear to have been limited by the growing cultural prohibition on quotation noted above: it may have been difficult on account of this readily to incorporate it *en bloc* into another context. Of course, its opportunities for replication in the minds of contemporary listeners are not restricted in this way.

Examples 1 (iii) and 1 (iv) show a longer Quotation than that in Examples 1 (i) and 1 (ii), the relationship here being between a Croatian folksong (after Grove 1896: 212) and the opening section of Beethoven’s Sixth Symphony (shown in Liszt’s (1922) transcription for piano). First identified by the ethnomusicologist, Franjo Kuhač (Samson 2013: 25), it is not possible to verify whether Beethoven took his material from a folk tradition or vice versa (Rosen 1997: 329, apropos of similar borrowings in Haydn), although Bartók was certainly sceptical of the latter scenario (Bartók 1976: 328). While this question is of concern to musicologists – and, in the case of possible borrowings by Austro-German composers from more ‘peripheral’ musical cultures, it has had controversial political implications (Schroeder 2009) – from the meme’s eye view survival by replication trumps such considerations of precedence. As with the Bach-Mozart Quotation in Examples 1 (i) and 1 (ii), the common material, bracketed, is a musemeplex consisting of several potentially independent unit-musemes.

3.2. Paraphrase

*‘typically involve different pitches but similar intervals paired with rhythmic freedom’. (See Table 1)*

Cope’s definition of Paraphrase requires some explication. From his Figures 3a and 3b – which show a resemblance between the opening melody of Stravinsky’s *Le sacre du printemps* and a Lithuanian folk melody (Cope 2003: 13) – it appears that what is meant is that a Paraphrase may change the key but will preserve most or all of the basic intervallic/scale-degree structure of the pattern, together with making some often significant rhythmic changes. Given that what is central to a museme’s identity is not its absolute pitch or key but its intervallic/scale-degree structure, the transposition is probably irrelevant. The changes definable as Paraphrase therefore inhere in the area of circumscribed intervallic/scale-degree modification, but to a significantly larger extent than in Quotation. These alterations are then sometimes augmented by association of the pitch museme with a different rhythmic museme (see Section 5.2.1 for a discussion of the relationship between pitch and rhythmic musemes). As with Quotation, often what is Paraphrased is not a unit-museme but a musemeplex, with one or more of its constituent musemes being subject to the changes described. In some such contexts, the changes are distributed among a number of musemes, preserving the identity of the whole, rather than being concentrated and localised in one
Example 1 (iii). Quotation: Anon: Croatian folksong

Example 1 (iv). Quotation: Beethoven arr. Liszt: Symphony no. 6 in F major op. 68 Pastoral (1808), I, bb. 1–4, 37–48

Allegro ma non troppo
and thereby significantly distorting it. In the latter situation, and in musemes of intermediate length, Paraphrase is often coterminous with museme mutation (see Figure 3).

Examples 2 (i) and 2 (ii) show such a musemeplex-Paraphrase, where Gluck bases the principal melodic phrase of his aria (transposed for ease of comparison) on the first reprise of the Gigue from J.S. Bach’s Partita in B, (Hayes 2014). While the surface figuration is modified, the harmonic structure is essentially preserved, apart from a mode shift (bb. 20 and 22 in the Gluck, from bb. 10 and 12 in the Bach) and alterations to the cadence (bb. 22f. in the Gluck, from bb. 12f. in the Bach). Here we have the ‘similar intervals paired with rhythmic freedom’ of which Cope speaks, and a passage for which Cope’s definition of Paraphrase shades into the combinatorially orientated one offered by Ratner, whereby ‘the basic harmonic and rhythmic structure is fixed; melody and texture are the variables’ (Ratner 1970: 35).

Examples 2 (iii) and 2 (iv) show transcriptions by Temperley (Temperley 2004: 323, Figure 4) of recordings of two versions of the opening of Scott Joplin’s ‘Maple Leaf Rag’, the first performed by the composer, the second performed by Jelly Roll Morton in 1938. Seen by Temperley as symptomatic of the move from ragtime to jazz (Temperley 2004: 322), which partly involved an increase in levels of syncopation, Morton’s changes inhere in subtle differences of melodic pitch and rhythmic placement within their (here unmarked) encompassing musemes, as well as in increased ‘swing’ which is intrinsically difficult to notate. This is shown on the examples, where boxed pitches in these phrases indicate pitch, rhythm, and onset-position invariants and where dotted boxes indicate pitch invariants which are displaced in terms of their onset-position. As with the Bach-Gluck relationship, these two passages represent a musemeplex-Paraphrase whose deep structure (as symbolised by the beamed elements of the examples) constitutes a Framework for the lower-level museme-sequence. Such Frameworks are balanced at the intersection of bottom-up and top-down forces: they are generated by the concatenation of a sequence of museme-alleles, yet they also regulate the nature and parataxis of this concatenation.
3.3. Likeness

‘have different pitches, intervals, and rhythms but have some underlying similarity such as overall likeness of directions or interval sizes’. (See Table 1)

Likeness arguably constitutes the loosest category of similarity. Indeed, for a Likeness to be asserted, a second level of memetic relationship sometimes needs to be invoked – that of the musico-operational/procedural meme (see Table 1). That is, a common musical strategy often affords the only way of linking two passages when their pitch and rhythmic content appears dissimilar. One common musico-operational/procedural meme is the idea of shared contour, or rather the mental representations underpinning several specific contour-schemes which are utilised in certain melodies. Folk music studies have arguably investigated this issue most thoroughly (Juhász and Sipos 2010), but it is certainly the case that melodies in Western common-practice music often conform to certain contour-archetypes, these perhaps relating to bodily image-schemata (Snyder 2000: 108–17; Shapiro 2011). Thus, two passages may be held...

Example 2 (iii). Paraphrase: Joplin: *Maple Leaf Rag* (?1917), bb. 0–4
Some of the Likenesses Cope posits might, however, more usefully be regarded as short, highly marked Quotations, such is their singularity (see Figure 4).

Examples 3 (i) and 3 (ii) show two forms of the trope Filius ecce patrem, the first from Aquitaine, the second from southern Italy (Treitler 1993: 492, Example 4). The broad arch of the melodies is certainly similar, but they differ in several significant details. The similarity of contour might be ascribed to the melodies sharing a common Framework, as defined by the notes connected by the upper bracket on Examples 3 (i) and 3 (ii). Structures connected by the lower brackets indicate hypothesised musemes — Treitler terms such note-groups ‘modules’ (Treitler 2007: 268) — not common to both passages (they are, nevertheless, Commonalities). But it is certainly problematic to make such a clear distinction between a quasi-middle-ground-level Framework and quasi-foreground-level Commonalities here, given the fusion of these proposed structural levels in the second half of the passages. Treitler associates these two tropes — together with another Aquitanian trope, a northern French trope and two others — as members of a ‘tune family’, a notion common in folk-music studies for such Likeness-related phrases which are assumed to constitute nodes in a highly complex network of transmission (Treitler 1993: 493; Cowdery 1984). Indeed — in an arguably false, anti-memetic dichotomy — he cautions that ‘[t]o account for the transmission of the trope Filius ecce patrem, the conception that its versions are all actualisations of a matrix … is preferable to the conception that the versions are related as variants of one another or of some hypothetical archetype’ (Treitler 2007: 269).

Examples 3 (iii) and 3 (iv) show a clear similarity of contour between two passages by Johann Strauss (the younger) and Mahler, indicated by the bracket above the staves of the two passages, but there is no Framework here comparable to that relating the trope melodies of Examples 3 (i) and 3 (ii). Other than the opening three note-museme (labelled ‘Museme x’ in the examples), the pitch content is dissimilar, but there is clearly a close general affinity of outline, embracing no fewer than eleven notes. It is not necessarily the case that Strauss’s tune is the direct and unmediated source of Mahler’s phrase: there may well be intermediate stages of replication; or it may even be completely unconnected to it. Nevertheless, the proposed relationship appears credible, given Mahler’s undoubted familiarity with the music of Johann Strauss, and his often ironic use of ‘Viennese’ idioms. So, while little concrete trace remains in Mahler of the melody which later appears in Strauss’s trio ‘So voll Fröhlichkeit’, it may have guided the symphony passage’s lower-level musemic sequence, perhaps acting as a selection pressure nudging Mahler to choose certain musemes over others.

3.4. Framework

‘involve the incorporation of interpolated notes so that potential similarity surfaces only after these notes are removed during analysis’. (See Table 1)

A hierarchic and reductionist view of musical structure underpins the notion of Framework (Schenker 1979; Narmour 1999), whereby one theme ‘lies across the superstructure’ of another (Cope 2003: 16). That is, above the foreground-level musemes of two passages sits a common shallow-middle-ground-level skeleton which, because it is replicated, is also a museme, albeit one at a more remote, ‘virtual’ level. Note that a memetic view is an a posteriori one, in which lower-level phenomena give rise to those at higher hierarchic levels. In three related scenarios, sometimes two such structures are (i) associated with different interstitial/generative musemes, in which case the Framework is loose and possibly difficult to perceive. However, they might instead (ii) be generated by similar interstitial/generative musemes, such that a shared middle-ground-level Framework arises from the repeat-conglomeration of a set of allelically equivalent
Example 3 (i). Likeness: Trope *Filius ecce patrem*, Aquitanian MS (eleventh century)

Example 3 (ii). Likeness: Trope *Filius ecce patrem*, Italian MS (eleventh century)

Example 3 (iii). Likeness: Johann Strauss II: *Der Zigeunerbaron* (1885), Overture, bb. 180–6

Example 3 (iv). Likeness: Mahler: Symphony no. 10 in F♯ (1910), I, bb. 16–20
In this sense, Framework elides with Paraphrase (intervallic/scale-degree similarity, often with rhythmic changes). At its most conformant, a clear Framework will inevitably result from (iii) the replication of a musemeplex, with two complexes made up of the same set of foreground-level musemes in the same sequence giving rise to a common higher-level structure. In this case, Framework elides with Quotation (often exact pitch and rhythmic duplication) (see Figure 5).

The notion of the Memesatz offers a means of understanding more remote Frameworks, including those spanning a whole movement (Jan 2010: 10–11); it might readily apply to the Joplin/Morton interwork in Examples 2 (iii) and 2 (iv), were the whole piece, and not just the opening bars, to be considered. It hypothesises that the structural nodes which articulate the essential navigational stages through a tonal movement (such as the pre-medial-caesura V:HC (Hepokoski and Darcy 2006: 24) and the turn to IV in the secondary development (Rosen 1988: 289)) are replicated from movement to movement. Although they are generally instantiated by different surface-level figuration, their basic configuration and function and the long-range relationships between them is essentially preserved and therefore so is the definition of the movement as an exemplar of a particular variant of sonata form (or whichever archetype is utilised).

The common galant schemata identified by Gjerdingen (2007; see also Byros 2009) often constitute Frameworks, encompassing other schemata and various non-schematic interstitial pitches; bb. 82–97 of the slow movement of Mozart’s Symphony in G minor K. 550 are a case in point (Gjerdingen 2007: 127, Example 9.20; Jan 2013: 164, Example 2). In Cope’s Figure 7 (Cope 2003: 16), however, which aligns the theme of the finale of Beethoven’s Eroica Symphony with a dance by Mozart, a Likeness not a Framework is read, presumably on account of the different underlying changing-note schemata, 1–7…7–1 in Mozart, 1–7…2–1 (the upper line of an ‘Aprile’) in Beethoven (Gjerdingen 2007: 122–3). A Paraphrase might also be understood here, emphasising the affinity of these three central categories.

Examples 4 (i) and 4 (ii) show a common intermediate-level Framework, the antecedent-consequent phrase. As indicated by the beamed and bracketed correspondences, within its regular periodicity, melodic symmetry and standard cadential gestures were established, such that it retained its integrity and coherence over a century, albeit latterly as a ghostly, historically pregnant, echo. Overlaid with Gjerdingen’s scale-degree symbology, where black-circled numerals indicate melodic degrees and white-circled numerals indicate bass degrees (Gjerdingen 2007: 20), Examples 4 (iii) and 4 (iv) show a ‘Meyer’ schema. Its proximity to the foreground in the Haydn passage – it comprises almost the entirety of these four bars – renders it arguably a pair (i–i–i–i; i–i–i–i) of Commonalities; but its expansion by interstitial pitches in the Leip/Schultze phrase – these constituting elements of (here unmarked) musemes – imparts to it the function of a Framework.

### 3.5. Commonality

‘typically involve patterns which, by virtue of their simplicity – scales, triad outlines, and so on – appear everywhere’. (See Table 1)

Commonalities are at once the most common type of museme and the most difficult to locate in a nexus of transmission relationships. For the reason given in the discussion of Quotation in Section 2, just because something is a Commonality does not mean that it is not a museme. In some cases, Commonalities shade into what Dennett terms ‘good tricks’ (Dennett 1995: 77–8). That is, they offer obvious
solutions to compositional problems and might therefore appear (as might indeed sometimes be the case) to have been invented rather than transmitted. As style shapes, Commonalities are the building blocks of the style structures which arguably constitute the majority of musemes (Narmour 1990: 34). Various configurations of style shapes may generate a set of style structures with the same shallow-middleground-level voice-leading structure, thereby giving rise to a collection of low-level, allelically related Frameworks. At the other extreme, certain Commonalities, such as Cope’s ‘triad outlines’, may themselves serve as Frameworks for fairly extended spans of music (see Figure 6).

As a three-note museme, the figure labelled ‘Museme y’ in bb. 3–4 of Example 5 (ii) is below the lower end of Miller’s continuum and it is difficult to trace an unambiguous source. Rather, it appears to have arisen ‘conceptually’, as a result of the operation of a musico-operational/procedural meme which regulates the voice exchange b₁–(d² implied)/d₁–b. It is a good way of filling these bars and harmonising the melody in contrary motion (another musico-operational/procedural convention of good counterpoint, of which Mozart would have been only too well aware), rather than being a pattern copied from a specific source context. Nevertheless, Mozart would presumably have been able to recall numerous such 2–1–7/2–1 musemes and their specific sources, such as that shown in Example 5 (i). In short, the fewer pitches in a museme, the more tangled and blurred the nexus of memetic transmission and the more likely such transmission was motivated by musico-operational/procedural factors.

Examples 5 (iii) and 5 (iv) show a cadential Commonality, labelled ‘Museme z’, which might be regarded as a ‘signature’ by Cope (1991). Usually short and often cadential in function, for Cope, signatures ‘can tell us what period of music history a work comes from’ (Cope 1998: 130). Nevertheless, if the chronological, geographical and cultural distance between these exemplars is representative of the distribution of other copies of this museme, it is arguably ahistorical (especially its final three notes, 3–2–1, which might be regarded as an independent sub-museme). If so, other copies might readily be located in various periods and places of music history.

3.6. Categorical Intersection

While the foregoing has discussed Cope’s categories as discrete, albeit linked on a loose continuum, it has also noted when certain cases of categorical intersection or hybridisation occur. These are summarised, and others hypothesised, in Figure 7, the numbers corresponding to the situations discussed after the figure. Perhaps unsurprisingly, Frameworks are implicated in several of these elisions.

(i) Quotation–Framework: a common middleground-level Framework may be generated by the repeat-conglomeration of the same set (a musemeplex) of foreground-level musemes (scenario (iii) in Section 3.4).

(ii) Quotation–Commonality: a small fragment may be marked in such a way as to impart to it the salience of a Quotation.

(iii) Paraphrase–Framework: a common middleground-level Framework may be generated by the repeat-conglomeration of a set of allelically equivalent foreground-level musemes (scenario (ii) in Section 3.4).

(iv) Likeness–Framework: a similarity of contour may result from two passages being partly anchored to a number of pitches common to their ostensibly different Frameworks.

(v) Likeness–Commonality: a similarity of contour may be built on one or more common stylistic fragment(s), at various structural levels. If the fragment(s) form(s) a Framework, this category intersects with (iv) above.

(vi) Framework–Commonality: the Framework pitches of a passage (possibly arising as a result of processes listed under (i) and (iii) above) may be equivalent to those of a common stylistic fragment, perhaps a scalic or triadic figure, situated at a higher structural level.

4. Quantification of Similarity: The Earth Mover’s Distance in Musemic and Cortical Space

While Cope’s five categories of allusion represent theoretical and analytical models intended to correlate broadly with listeners’ perceptions, his Sorcerer program necessarily implements a more objective, algorithmic approach to similarity detection than is evident in the responses of human subjects. I turn here to consider such approaches in more detail and examine how they might relate to various aspects of similarity – psychological, evolutionary, and neurobiological – of relevance to memetics.
Example 4 (i). Framework: Mozart: Piano Concerto in C major K. 415 (387b) (1783), III, bb. 1–8

Example 4 (ii). Framework: Tchaikovsky: *Nutcracker Suite* op. 71a (1892), Overture Miniature, bb. 1–8
Example 4 (iii). Framework: Haydn: Minuet from Divertimento in C major Hob. XIV: 10 (c. 1760), bb. 1–4


Figure 6. Commonality
Example 5 (i). Commonality: Haydn: Symphony no. 60 *Il distratto* (1774), I, bb. 195–7

Example 5 (ii). Commonality: Mozart: Symphony no. 41 in C major K. 551 *Jupiter* (1788), I, bb. 1–4


![Music notation](image)

4.1. Models of Musical Distance and Weight

As noted in Section 1, various computational approaches have been developed in order to quantify similarity in music. Some of these aim to model perception, in that two passages, ranked according to their underlying algorithms as closely related, are shown to be perceived as such by listeners. Müllensiefen and Frieler (2004; see also 2006) evaluated some forty-eight similarity-detection algorithms, comparing them with the responses of listeners in psychological tests of melodic similarity. Their findings suggest that some of the most robust metrics of melodic similarity are of the ‘edit-distance’ type (Müllensiefen and Frieler 2004: 168), whereby the cost of moving from one pattern form to another is quantified. A well-established example of this type is the metric proposed by Damerau and Levenshtein (Levenshtein 1966; Orpen and Huron 1992). This assesses the notional costs, according to some predetermined scale of values, of the operations of insertion, deletion, and replacement, by means of which a source text is transformed into a target, or a target is understood to be related to a source.

A related approach, the ‘Earth Mover’s Distance’ (EMD) metric, first developed in the context of image retrieval research (Rubner et al. 2000) and then applied to music (Typke et al. 2003; Wiering et al. 2004; Typke 2007; Typke et al. 2007),

\[
\text{determines the minimum amount of work that is needed for converting one set of weighted points into another. The required work grows with the amount of weight that needs to be moved to different positions, and with the distance over which the weight needs to be moved (Typke et al. 2007: 154–5).}
\]

Put more simply,

\[\text{[o]ne pattern ... is represented as heaps of earth, the sizes of which correspond to the weights of the dots; the other pattern ... as holes with a certain capacity, likewise corresponding to the dots’ weights. The task is to fill the holes with as little effort (that is, ground distance times weight) as possible (Wiering et al. 2004: 117).}\]

The EMD is defined as:

\[
EMD(A, B) = \frac{\min_{F \in \mathcal{F}} \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij} d_{ij}}{\min(W, U)}
\]
A (source) and B (copy) are sets of weighted points. The symbol $F$ represents the set of all possible flows that would convert A into B. Every flow element carries a weight of $f_{ij}$ over a ground distance $d_{ij}$ from one point in A to one point in B. The number of points in sets A and B are $m$ and $n$, respectively. The sums of weights in sets A and B are W and U, respectively (Typke et al. 2003: 109; 2007: 155).

One way to illustrate the EMD is by a ‘weighted-point set’ diagram, which shows the source set of points (notes), the target set, and the weight moved between them (Typke et al. 2007: 155–7). The upper part of Figure 8 represents the opening of the C major Fugue from Book I of J.S. Bach’s Das Wohlteperirte Clavier. In b. 17, the subject returns in D minor. If this entry were transposed to the starting pitch of b. 1, then the e$\flat$ and a$\flat$ quavers (eighth notes) of the fourth beat would become e$\natural$ and a$\natural$ respectively, as represented in the lower part of the figure. This change is represented by the movement of the two points. While the weighted-point set diagram is a useful way of visualising the EMD in musical contexts, another approach will be considered in Section 4.4.

‘Weight’ and ‘distance’ may be conceived in terms of movement in Euclidean space (Typke et al. 2007: 156, 176–7), and can be quantified in a variety of ways. For weight, the most straightforward quantification is rhythmic: the longer the note’s value, the greater its weight. But other quantifications are possible: one might adjust a rhythmic weighting with a metrical factor, giving notes on strong beats greater weight than those on weaker beats. Alternatively, or perhaps in conjunction with rhythmic-metric weightings, the tonal stability of a pitch within its prevailing key system (its ‘key-profile’) might be employed (Krumhansl and Kessler 1982; Krumhansl 1990; Temperley 2001: 180; Typke et al. 2007: 176). That is, moving a relatively stable scale degree (such as 1 or 5) is taken to involve greater effort than moving a relatively unstable one (such as $i$ or $7$): the relatively stable degree is more resistant to change than the relatively unstable degree. A related metric, when used as a basis for EMD weightings, might be the distance along the ‘line of fifths’ between two notes (Temperley 2001: 118, Figure 5.4). Each movement from note to note involves a corresponding movement along the line of fifths, which is naturally greater for some motions than for others. Diatonic notes are closely arranged along the line (Temperley 2001: 127, Figure 5.11), whereas movement from a diatonic to a chromatic note inevitably results in larger segments of the line being traversed.
Figure 8. Weighted-point set diagram for J.S. Bach, Fugue in C Major BWV 846 from Book I of *Das wohltemperirte Clavier* (1722), bb. 1, 17.
Yet another metric might be Jan’s i-rp (Jan 2007: 129–33, Table 4.1; see also Yazawa et al. 2013) at a given note or of a subgroup of notes. Here, moving in a way which violates an implication (such as a continuing Process, or an implied Reversal (Narmour 1990: 99–100, 150–1)) imparts to the ‘deviating’ note (the third in a three-note implicatve structure) a greater weight than one which realises the implication. In this way, g1 in the sequence c1–d1–g1 would be more heavily weighted than e1 in the sequence c1–d1–e1, for the g1 would constitute a violation of Process (it would need more ‘energy’ to deflect its greater weight away from its expected trajectory), whereas the e1 would constitute a realisation. In short, the i-rp is intended to offer a way to quantify and compare the ‘tiny cognitive “jolts” to the neuronal electrical system governing our subconscious cognitive expectations’, which afford to musical patterns their differential salience (Narmour 1990: 138). These various approaches will be explored in Section 5.2 as part of a synthesis of Copean, memetic, and transportational perspectives on similarity.

4.2. The EMD and Psychological Aspects of Similarity

The EMD accords well with psychological, evolutionary, and neurobiological perspectives on pattern perception and transmission/mutation, and these will now be considered in turn. From a psychological standpoint, one might hypothesise that the difference-distance between two pattern-forms is proportional to the effort required in cognition to relate them. The connection between two closely related forms, whereby a low-value EMD connects them, is therefore easier to cognise than that between two more divergent forms, which will be separated by a high-value EMD. The EMD is thus both a measure and a predictor of perceptual-cognitive similarity and therefore of ease of recognition. In this context, the EMD might also be regarded as an index of the perceptual-cognitive salience of the copy museme: how singular or striking the copy museme is, when cognised in comparison with the memory of its source, might be hypothesised to be directly proportional to the EMD value, the latter being assigned in cognition to the copy form.

4.3. The EMD and Evolutionary Aspects of Similarity

Two replicators (genes or memes) related homologically can be understood as occupying proximate positions in a Euclidean space – a multidimensional genetic/memetic hypervolume (Dawkins 1991: 67–8; Jan 2007: 197–201). The operation of the Darwinian evolutionary algorithm has shifted one replicator (or rather a modified form of it) from its initial position to that of the other. Thus, in evolutionary terms, transportation distance is effectively ‘mutation distance’. For two more widely spaced replicators, a web of transmission might be traced which connects them by means of stepping-stone intermediate stages. In such cases, the cumulative work needed to move from the source to the most remote copy is substantial, although that required to move between an adjacent pair of the individual steps which connect them is relatively small. A psychological equivalent of this phenomenon is presumably operative.

This idea relates to Dawkins’ distinction between ‘single-step’ and ‘cumulative’ selection (Dawkins 1991: 45), itself analogous to what Dennett terms the difference between ‘skyhooks’ and ‘cranes’, respectively (Dennett 1995: 73–5). Single-step selection, sometimes associated with the evolutionary theory of ‘saltationism’ (Dawkins 1991: 230), is unlikely in direct proportion to the distance to be jumped across the hypervolume; whereas single-step selection is capable of traversing great expanses, provided of course the individual steps are manageably small. Naturally, the greater the total distance traversed between the initial and terminal points, the harder it is to detect (or defend) an evolutionary/homological connection, unless the intermediate stages, and the connections between them, are readily evident.

4.4. The EMD and Neurobiological Aspects of Similarity

The EMD relates elegantly to certain theories of neuronal information encoding, specifically that proposed by the American neuroscientist, William Calvin (1995, 1996, 1998). In his Hexagonal Cloning Theory (HCT), the attributes of a discrete object in perception are encoded by co-resonating triangular arrays formed from interconnected ‘minicolumns’ of pyramidal neurons in the cerebral cortex. These arrays are grouped together, via ‘interdigitation’, into plaques of abutting hexagons – ‘the minimal Hebbian cell-assembly … [with] 0.5 mm between parallel sides’ (Calvin 1996: 45). Each hexagon encompasses a set of coordinated attributes, such as those defining a discrete, gestalt-partitioned group of pitches and rhythms – a museme, if it is replicated. In Calvin’s view, Darwinian processes regulate the copying of the hexagons across the surface of cortex, the ‘winning’ configuration being that conforming most closely to the incoming data or remembered pattern (for a fuller discussion of the application of Calvin’s theory to musemes, see Jan 2011). The HCT arguably offers a robust neurobiological explanation of the museme in its most fundamental (memotypic) form, and one which goes beyond earlier work in this field, such as that of Delius (1991: 82–3), with its discussion of ‘synaptic constellations’. 
A given area of cortex can support numerous overlapping hexagons as a result of the embedding of multiple ‘attractors’ in the cortex. That is, there are notional ‘basins of attraction’ which strengthen certain connections (often the ones we are most frequently exposed to) and which hardwire certain patterns into the connectivity as long-term memory. These basins capture incoming sensory data, sometimes bending it towards the configuration of remembered information, which exists as established ‘spatiotemporal firing patterns’ (SFPs). Calvin (1996: 107) likens our numerous overlapping SFPs to the layers of fish in the Japanese delicacy, sashimi.

More recent work in this area has concerned spatial representation using the ‘grid cells’ of the entorhinal cortex (Shrager et al. 2008; Burak and Fiete 2009; Stensola et al. 2012; Killian et al. 2012). These are thought to be implicated in positional location in two-dimensional space and, like certain other brain systems, map incoming topographic data systematically onto cortical neurons. While such studies tend not to make explicit reference to the HCT (but see Garliauskas and Šolliūnas 2000), and while some even considerably predate Calvin’s work (O’Keefe and Dostrovsky 1971), the cells they describe also form triangular arrays which are similarly grouped into hexagonal plaques (Fuhs and Touretzky 2006: 4269, Figure 2). Outlining an experiment on monkey visualisation, Killian et al. observed that

\[\text{individual neurons were identified in the primate E[ntorhinal] C[ortex] that emitted action potentials when the monkey fixated multiple discrete locations in the visual field in each of many sequentially presented complex images. These firing fields possessed spatial periodicity similar to a triangular tiling with a corresponding well-defined hexagonal structure in the spatial autocorrelation…. These spatial representations may provide a framework to anchor the encoding of stimulus content in a complex visual scene. [These] results provide a direct demonstration of grid cells in the primate and suggest that EC neurons encode space during visual exploration, even without locomotion. (Killian et al. 2012: 761)\]

Thus, the field of research these studies describe, in conjunction with Calvin’s work, may be argued to reveal a broadly unified mechanism for the encoding of certain types of sensory data. I argue below that this systematic mapping of real-world space to the geometry of the cortical surface might be extrapolated to encompass the mapping of more abstract spaces, specifically those engendered by pitch.

Two types of earth moving appear to be implicated in the operation of the HCT, these being represented abstractly in Figure 9. Each row of dots represents a museme, its five elements (a–e) constituting a weighted-point set, and the horizontal and diagonal arrows represent movement of neuronal activation from one attractor basin to another. The first type of earth moving, what might be termed ‘intra-museme earth moving’, concerns the note-to-note progression along the pitches of a museme in real-time perception and cognition. While the EMD has no inherent notion of note-to-note progression (weight simply flows over minimum distances), and while weight may move in non-intuitive patterns between points in order to satisfy the minimum low constraint\[^{24}\], the single unfolding line traced by a melodic museme might be regarded as the movement of a certain weight over a certain distance. This movement involves taking the weight of the first pitch and then expending energy in order to lift it out of its current attractor basin into the gravitational pull of the second attractor basin, and so on, until the end of the museme is reached. In this sense, the metaphor for the EMD described in Section 4.1 appears close to reality at the cortical level. The total effort expended in moving the melodic ‘cursor’ from attractor to attractor constitutes the total EMD of the museme, and might also be regarded as an index of perceptual-cognitive salience (Section 4.2).

As for the second type, what might be termed ‘inter-museme earth moving’, new basins of attraction are formed in competition with established ones when a mutans is mutated. For the mutans to be encoded, it must excavate a new basin of attraction around any altered (copy-specific) pitch(es), in order to divert the SFP away from the original (source-specific) pitch(es) and towards the new one(s). The latter is/are symbolised in Figure 9 by a superscripted letter (a′–e′) arrived at via a vertical red arrow. This, of course, is also a form of earth moving, in that the earth is being cleared away from around the new pitch in order to create a fresh basin. This earth does not necessarily end up filling the source pitch(es)’s basin(s), for the source may continue to exist in memory (and of course one or more of the source’s pitches may remain in the copy museme, albeit potentially in a different sequential position within the museme). But, if the copy form comes to overshadow the source, memotypically and/or phenotypically, then the earth might be regarded as effectively occupying the site of the source museme’s deleted pitch(es). While this article is primarily concerned with inter-museme earth moving, because of its focus on similarity and mutation, Section 6 will briefly discuss the intra-museme type.
One of the salient features of the auditory nervous system is that a tonotopic organisation exists from the earliest level of the periphery, at the basilar membrane, to many fields within the auditory cortex (Zatorre 2003: 233; see also Stainsby and Cross 2009: 48–9; and Jan 2011: Sec. 4.1.2). As suggested above, it appears reasonable to infer that the surface of the auditory cortex (for structural diagrams, see Brattico 2006: 15) is ‘tuned’ to pitch (more properly, frequency) in a broadly systematic way, but with sufficient pitch-receptor repetition and interdigitation to allow hexagons to encompass the same pitch class in different octaves and in different topographical configurations. This inference is made by analogy with the positional location mapped by entorhinal cortex grid cells and also with the spatial orientation of neurons in the visual cortex (Braitenberg and Braitenberg 1979; Garliauskas and Šolijonas 2000: 404, Figure 3).

Inter-museme earth moving is depicted in Figure 10, which again represents the opening of Bach’s C major Fugue but this time reimagined in terms of the HCT. Thus, while Figure 8 is a high-level abstract representation, Figure 10 is rather more low-level and concrete. Triangles shown in red are active (firing) and those shown in blue are inactive (silent); dotted red triangles represent mutated pitches, the glowing red circles around them representing newly established basins of attraction. Activated arrays therefore constitute the cortical equivalent of weight, and the transfer of activation across the cortical surface (driven by cycles of neurotransmitter production) represents the cortical equivalent of its movement. The various quantification schemes discussed in Section 4.1 appear to be supported by certain intra-brain mediating factors which impinge upon the perception and conception of pitch weight. These include representations of pitch stability, thought to be controlled by hexagons located in the frontal cortex, which are connected to the pitch-encoding hexagons in the auditory cortex by long-range brain connections termed ‘faux fax’ links by Calvin (1996: 125, 131).

The triangles in Figure 10 are arranged, speculatively, in the manner of a Riemannian Tonnetz, specifically the ‘Oettingen/Riemann Parsimonious Tonnetz’ given by Cohn (1997: 15, Figure 9a). This is on the basis that, while the detailed implementation of pitch encoding in the auditory cortex remains unclear, ‘one of the salient features of the auditory nervous system … is that a tonotopic organisation exists from the earliest level of the periphery, at the basilar membrane, to many fields within the auditory cortex’ (Zatorre 2003: 233; see also Stainsby and Cross 2009: 48–9; and Jan 2011: Sec. 4.1.2). As suggested above, it appears reasonable to infer that the surface of the auditory cortex (for structural diagrams, see Brattico 2006: 15) is ‘tuned’ to pitch (more properly, frequency) in a broadly systematic way, but with sufficient pitch-receptor repetition and interdigitation to allow hexagons to encompass the same pitch class in different octaves and in different topographical configurations. This inference is made by analogy with the positional location mapped by entorhinal cortex grid cells and also with the spatial orientation of neurons in the visual cortex (Braitenberg and Braitenberg 1979; Garliauskas and Šolijonas 2000: 404, Figure 3).

Twenty-four triangles are shown in Figure 10. They cover the pitch range a₁–c₅ and are arranged across a field in which horizontal motion traverses a minor third, vertical motion a major third, diagonal motion SW–NE a perfect fifth, and diagonal motion SE–NW a semitone. The firing order of arrays is indicated by the boxed numbers adjacent to the triangles’ pitch designations. These also show the duration of the impulse (note length), using traditional western rhythmic symbols. The museme mutation of b. 17 is represented by the arrows pointing from the source S/T (spatial (pitch)/temporal (sequence/rhythm)) coordinates to those of the copy.
Figure 10. Hypothesised cortical encoding of J.S. Bach, Fugue in C Major BWV 846 from Book I of *Das wohltemperirte Clavier* (1722), bb. 1, 17

The total EMD value here, 0.656414, is calculated using Temperley's Krumhansl-based weightings (see Section 5.2 for details). How is this value to be interpreted as part of a memetic analysis? The absolute value of the EMD depends upon the particular parameters utilised in its calculation and the specific weightings applied to them. Moreover, the EMD depends in large part upon the number of elements in a museme. Given this, EMD values are perhaps best employed as comparators rather than as numbers with intrinsic significance. That is, retaining the same weightings for all calculations allows a number of musemes, in both their source and copy forms, to be compared, using the EMD as an index of relative mutational distance (Section 4.3). Such comparisons may be applied, to mention just two approaches, within a pair of musemes (looking solely at the source and copy in a hypothesised evolutionary relationship) and between museme pairs (looking at how the mutational distance between one pair of musemes compares with that between another pair). I return to this issue in Section 6.

### 4.5. Summary of the EMD in Relation to Memetic Phenomena

Adapting the format of Figure 1, Figure 11 summarises the relationships between the EMD and various psychological, evolutionary, and neurobiological phenomena.
Figure 11. Summary of relationships between EMD and various psychological, evolutionary and neurobiological phenomena

<table>
<thead>
<tr>
<th>Psychological</th>
<th>Evolutionary</th>
<th>Neurobiological</th>
</tr>
</thead>
<tbody>
<tr>
<td>perceived dissimilarity</td>
<td>evolutionary disconnection</td>
<td>extent of neuronal rewiring</td>
</tr>
<tr>
<td>conceived dissimilarity</td>
<td>mutational distance in a multidimensional memetic hypervolume</td>
<td>number of new basins of attraction</td>
</tr>
<tr>
<td></td>
<td>likelihood/degree of saltation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>prevalence in museme-pool</td>
<td></td>
</tr>
</tbody>
</table>

Association between antecedent Museme $x$ and its potential consequent Museme $x^1$:

5. A Case Study: Beethoven, Piano Sonata in B♭ major op. 106

_Hammerklavier_ (1818), IV, bb. 111–27

This section identifies seven musemes in a passage from a Beethoven sonata and suggests potential source coindexes (i.e., matching, homologously related forms) to them from works of Beethoven's immediate and more remote predecessors. It then classifies the source and copy forms according to Cope's categories in order to understand how copy musemes of various Copean types are integrated in a new idiostructural context. It then applies the EMD calculus to the musemes in an attempt to measure the psychological, evolutionary, and neurobiological distance between the hypothesised source and the Beethovenian copy according to the various weighting schemes posited in Section 4.1. In this way, the relationship between museme form and salience may be explored and quantified. In addition to developing a methodology for such an application of the EMD (which can be refined in future research by, for instance, the addition of other weighting schemes representing more subtle indices of salience), the analysis will give rise to data which can be used (among other things) for assessing the probability that two given patterns are homologically related, and for feeding into the rather more ambitious project of charting the museme-pool outlined in Section 6.
Example 6. Musemes in Beethoven, Piano Sonata in B major op. 106 *Hammerklavier* (1818), IV, bb. 111–27

![Musemes in Beethoven's Piano Sonata in B Major](image)

(f) Beethoven: Piano Sonata in B major op. 106 (*Hammerklavier*) (1818), IV, bb. 16–20 (e), plus associated sketches (a–d)

*fuga a tre voci con alcune licenze*

16

![Motive a](image)

Motive b

<table>
<thead>
<tr>
<th>Motive a</th>
<th>Motive b</th>
<th>Motive c</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Motive a" /></td>
<td><img src="image" alt="Motive b" /></td>
<td><img src="image" alt="Motive c" /></td>
</tr>
</tbody>
</table>

![Motive c](image)
Example 6 (cont.). Musemes in Beethoven, Piano Sonata in B♭ major op. 106 *Hammerklavier* (1818), IV, bb. 111–27

(iii) J.S. Bach: Fugue in D♭ minor from Book II of *Das wohltemperierte Clavier* (c. 1744), bb. 1–3

(iv) Haydn: String Quartet in B♭ major op. 55 no. 3 (1788), IV, bb. 9–4

(vii) Beethoven: Piano Concerto in B♭ major K. 595 (1791), I, bb. 175–5

(viii) Beethoven: Piano Concerto no. 4 in G major op. 58 (1808), I, bb. 187–8
Example 6 (cont.). Musemes in Beethoven, Piano Sonata in B, major op. 106 *Hammerklavier* (1818), IV, bb. 111–27

(b) J.S. Bach: Fugue in F minor from Book II of *Das wohltemperierte Klavier* (c. 1744), bb. 16–20

(iv) Mozart: Piano Concerto in C major K. 467 (1785), I, bb. 1–4

(iii) Mozart: Piano Concerto in B minor K. 466 (1785), I, bb. 231–3
Example 6 (cont.). Musemes in Beethoven, Piano Sonata in B, major op. 106 *Hammerklavier* (1818), IV, bb. 111–27
5.1. Memetic Analysis and Hypothesised Source Coindizes

In a fugue rich in demonstrations of contrapuntal virtuosity and rigour, the first of three main rhetorical-structural pillars is the presentation of subject 1 in augmentation, inversion, and stretto in bb. 94ff. Briefly, the subject appears in augmentation in b. 94 in the middle voice, with the countersubject appearing two bars later in the bottom voice. This leads to a tonal answer at b. 110 in the top voice followed, at b. 113 in the bottom voice, by an augmented version of the subject in inversion. The juxtaposition creates a stretto which, at b. 116, disintegrates into semiquaver (sixteenth-note) arpeggios and then registrally isolated trills. The intensity finally dissipates, leading into the next part of the fugue, an episode in A major, and a return to a more normative texture. The most tumultuous part of this passage is shown in Example 6 (x). The passage, like many of Beethoven’s most audacious, is based upon commonplace musemes: compared with the works of Beethoven’s forerunners and contemporaries, their mutation and arrangement is radically different; but the underlying patterning remains remarkably similar to that of music written often many decades earlier.

It is important to note that musemic replication is constrained by the ‘environment’ of fugue differently from the environment of sonata style. The most important of these constraints is the operation of specific musico-operational/procedural memes governing aspects of melodic museme treatment. The former regulate such local procedures as augmentation, diminuation, inversion, and retrogression; and more extended procedures such as exposition, re-exposition, and stretto. Apart from their circumscribed use in the sonata-form development, the impact of these musico-operational/procedural memes is relatively limited outside fugal (and serial) textures. In fugue, however, they tend to privilege the intra- over the inter-work dimension. Thus, a figure appearing in a fugue is, ceteris paribus, more likely to be an idiostructural derivative of the subject(s) and/or countersubject(s) than to be an import from the dialect – although (as a memetic orientation would insist) the influence of the inter-work dimension is never absent. Nevertheless, the two dimensions interact, in that idiostructural mutation may be skewed by memory of extra-work musemes: this recollection potentially directing the reshaping of the internal in the image of the external.

At the beginning of the passage (bb. 111–162) the figuration is based upon what Busoni terms ‘motive b’ of the fugue subject, labelled ‘Museme b’ in Example 6 (ix) (e), and its variants (Busoni 1894, Supplement, Appendix 3: 195). Essentially a grouping of six or seven notes (semiquavers in the original form of the subject, quavers in this augmented passage), the six-note form constitutes one of the most common musemes in late-eighteenth and early-nineteenth-century music. It exists in various forms which are distinguished according to their scale-degree sequence and placement of harmonic and non-harmonic notes. The distinction between perceiving a six- or a seven-note group hinges on the status of the seventh pitch, whose change of direction (a Retrospective Registral Reversal (VR) (Busoni 1894: 335)) and position after the bar line in Beethoven’s subject potentially sets it apart from the first six. Nevertheless, the longer duration of the seventh pitch, which creates a subsequent segmentation boundary by means of ‘durational interference’ (Narmour 1989: 45–6), and the rests before the first and after the seventh pitches (in the original form at bb. 17–18), are strong gestalt grouping cues, binding all seven pitches together into a single perceptual-cognitive unit.

A potential source coinindex from Haydn is shown in Example 6 (v), in which the Retrospective Registral Reversal is not associated with durational interference, resulting in six-, not seven-note groupings. But the contour of the bracketed segment is identical to that of Beethoven’s Musemes b and b’ (and thus the whole picks out the descending third-progression which is so prevalent at the local and global levels of Beethoven’s movement and indeed the sonata as a whole (Rosen 1997: 407–34)), and the shared key of the passages reinforces the posited evolutionary connection. Further evidence that the Haydn passage may have been significant can be found in the sketches for the fugue subject, through which any source will have been filtered. Example 6 (ix) (a–d) shows some of these sketches which, despite alternating between 3/4 and 4/4, cleave to the descending third-progression articulated by the six-note figures, albeit with different continuations (Nottebohm 1887: 136). Significantly, Example 6 (ix) (b) continues to follow the broad contour of Haydn’s theme beyond the segment possibly adapted for the fugue subject, as shown by the asterisks added to the sketch and to Example 6 (v). The same affinity (similarly indicated) is evident, albeit to a lesser extent, in Example 6 (ix) (d).

In the middle voice of b. 111, Beethoven uses a five-note form of this museme, Museme b”, wherein the reversal comes after note 4. The expected continuation is shown in bracketed small noteheads with a dotted beam. The two anticipated middle-voice pitches, c and b,, appear to have lingered in Beethoven’s memory for, as shown by the arrow, they appear shortly afterwards, as the lower-voice trilled octaves in bb. 112–14, marked ‘Museme c’ (idiostructurally, an inversion of the subject’s ‘motive a’). The lower-voice shadow of...
Museme $b^2$ might similarly be understood to motivate the middle-voice transposed copy of the lower-voice Museme $c$, again shown by the arrow, the crossing of these arrows representing what might be termed an ‘implication exchange’. In this way one museme, or rather the expected but missing pitches of a museme, triggers another and impels it into existence. Museme $c$ is related to an allele-class of cadential musemes, one member of which is especially common at structural points in Mozart’s piano concerto; a source coindex is shown in Example 6 (vii). A pattern labelled ‘Museme $a’ appears at the end of Museme $b^2$. This bears a resemblance to a segment of the subject from J.S. Bach’s Fugue in $D_7$ minor from Book II of Das wohltemperirte Clavier, shown in Example 6 (ii). Here, the entry of the answer in b. 3 acts as a segmentational force, represented by the dotted vertical line, partially obscuring the upper voice after $e_2^1$ and therefore – in a possible explanation for the segmentation between Beethoven’s $c^2$ and $g^2$ at b. 112$^{23}$ – to some extent overcoming its sense of continuity to $d_1^1$.

In conjunction, and from a bottom-up perspective, the foreground-level musemes of bb. 111–4 generate a mutated version of the Prinner schema (6/4–5/3–4/2–3/1). Gjerdingen’s scale-degree symbology is overlaid on Example 6 (x), and a source coindex from Beethoven’s Piano Concerto no. 4 is shown in Example 6 (viii). In memetics, the bottom-up always exists in a dialectical interplay with the top-down. From the latter perspective, the middleground-level Prinner — in association with the aforementioned fugue-regulating musico-operational/procedural memes — might be said to have controlled the assemblage of the foreground-level musemes at this point.

The process of delayed musemic completion linking Musemes $b^2$ and $c$ also appears to have been operative in b. 116. The expected continuation of the upper-voice Museme $b^2$ and its middle-shadow involves three pitches, $e_1^1$, $b_1^1$, and $d_1^1$, which appear, as $e_2^2$, $b_2^2$, and $d_3^2$, in the arpeggio at b. 116$^2$ (the $c^2$ also implied is realised as the bass note at this point). Opening the next phase of the passage (bb. 116$^3$–8$^3$), this semiquaver figuration is generic to the concerto (for Ratner, it would constitute an example of the ‘brilliant style’ (Ratner 1980: 19)), and a particularly close source coindex might be found in the development section of Mozart’s Piano Concerto K. 466, shown in Example 6 (iii) as Museme $d$, where the local tonal context is analogous to Beethoven’s$^{34}$. That is, both passages progress from a retrospectively understood iv$^3$ in F minor (Mozart, b. 231$^2$; Beethoven, b. 116$^{1–2}$) to its V$^3$ (Mozart, b. 232; Beethoven, b. 116$^3$). Moreover, while the pitch sequence of Beethoven’s figuration, Museme $d^2$, is not identical to Mozart’s, the pitch content of the bracketed sections is, being a filling-in of the octave from $d_1^3$–$d_2^2$ with an arpeggiated diminished seventh chord.

The following material (bb. 118$^3$–27) is underpinned by that most ubiquitous of eighteenth-century harmonic progressions, the circle of fifths. It rotates through C (b. 117), F (b. 120), B (b. 123), E (b. 125), to A$\flat$ (b. 130), in readiness for the inverted return of the episode which first appeared in G, at b. 85. Countless examples of this progression, marked ‘Museme $e’$, might be found in music before Beethoven. In baroque fugues, a source coindex appears in bb. 16–20 of J.S. Bach’s Fugue in F minor from Book II of Das wohltemperirte Clavier, shown in Example 6 (i). Of course, Beethoven could have had any number of similar passages in mind or, perhaps more likely, drawn upon a mental abstraction of the museme allele-class formed by absorbing numerous examples. In most contexts, including the Bach example and that in Beethoven’s passage, the progression is a shallow-middleground-level harmonic museme generated, as always, by the conglomeration of foreground-level musemes.

Of the latter, the line traced in bb. 117–20 is a registraly expanded version of a familiar $5–7–2–4$ dominant-seventh-based melodic museme found in such examples as the opening theme of Mozart’s Piano Concerto K. 467, shown in Example 6 (iv) and marked ‘Museme $f’$. In this context, a musico-operational/procedural meme is deployed to mutate a familiar close-position scale-degree sequence by extending it over four octaves. Both occurrences of this museme, ‘Museme $f’$, are followed by one, ‘Museme $g’$, normally found in cadential positions in which a dominant seventh is suspended over a tonic pedal. A source, ‘Museme $g’$, is shown in Example 6 (vi), from Mozart’s String Quartet K. 575. As Beethoven’s passage reaches the dominant of A$\flat$, another variant of Museme $fff$, labelled ‘Museme $f’$, might be read in bb. 124–7, the ninth $f’$ substituting for the expected seventh $d_2^7$ in b. 127.

5.2. Earth Mover’s Distance Analysis

The musemic relationships in Example 6 are summarised in Example 7. Copy musemes in Example 6 (x) are transposed to C major or C minor in Example 7 and are shown below their source coindices, similarly transposed. This equivalence not only facilitates comparison but is also the basis of the EMD calculation which, as suggested in Section 3.2, should not consider differences in pitch resulting from museme transposition as evolutionarily significant. To the right of each museme in Example 7 are shown the values from which the EMD
is calculated in the format \((W,X)\), where \(W\) denotes pitch (as MIDI pitch number; \(c_1 = 60, c_2 = 61, b = 59\), etc.) and \(X\) denotes onset point (starting from the beginning of the first bar of the museme, measured to the nearest \(d = 1.0\), and continuing consecutively through the museme). In this way, the \(X\)-value dimensions of the ground-distance space are standardised by the use of common, low-value musical units. One such \((W,X)\) grouping represents one note of the museme, the set of notes making up the museme being enclosed by an outer pair \(\{\}\). The musemes are also represented as Calvinian hexagons in cortical space in Figures 12–18 in order to represent visually the choreography of mutation at a neurobiological level.

As discussed in Section 4.1, various weighting schemes may be used to calculate the EMD values\(^4\). The output from five of these is shown in columns (i)–(v) of Tables 2–8, their average values are shown in column (vi), and their normalised values (average/number of museme elements) are shown in column (vii). Naturally, when making comparisons between EMD values for different musemes, it is necessary to compare values for the same weighting scheme (comparing the value in column (i) for one museme with that in column (i) for another, etc.). This is because (i) the differences in magnitude of the various weighting schemes, while internally consistent, is arbitrary; and (ii) the numbers are simply larger for certain weighting schemes than for others, which means that those weights have a greater influence on the computed EMD with respect to the ground distance covered. The values in column (vi), being an average of the columns (i)–(v) values and being standardised through division by the number of museme elements, perhaps give the most meaningful comparisons. It might be argued that these are ‘apples and oranges’ metrics: the absolute rhythmic length of a note is different in kind to the perceived relative stability of a tonal scale degree. Nevertheless, they offer a variety of perspectives on the phenomena in question whose varied patterns of congruence and non-congruence, while not all addressed or explicated here, are certainly interesting.

The metrics employed are rhythmic weighting (where \(d = 1.0\); Krumhansl’s and Kessler’s original key-profile (Krumhansl and Kessler 1982; Krumhansl 1990: 79–80), hereafter ‘Krumhansl’\(^4\); Temperley’s revision of this profile (Temperley 2001: 180)\(^4\); Temperley’s line-of-fifths metric\(^5\); and Jan’s i-r metric\(^6\). These metrics were selected in order to offer a means of measuring a range of different attributes of music which impinge upon its perceptual-cognitive salience and which can be utilised in EMD calculations for the purpose of comparing the salience of various forms of a museme.

To explain the i-rp in more detail, an i-r structure is a five-element unit, consisting of three pitches (initial, medial, and terminal) interleaved with two intervals (\(\cdot \times \gamma \cdot\)). An i-rp value is held to apply to the whole structure. In the case of several interlocking i-r structures (such as a museme of five pitches, which would therefore encompass three i-r structures), the total i-rp is the sum of the constituent i-r structures’ individual i-rp values\(^6\). For the purposes of the EMD calculations, however, the i-rp is taken to apply to the terminal pitch of each i-r structure: the value is assumed to reside in that pitch. The first two pitches of the first i-r structure, which are unweighted on this basis, are each given a notional value equal to the average of the i-rp values of the other pitches (they might alternatively have simply been weighted at 1.0). In the case of Example 7, Museme a, the four i-r structures (respectively (VR), (R), P, and P)\(^4\) give rise to a total i-rp value of 10.9, the individual pitches of the museme being weighted as 2.7, 2.7, 1.5, 5.3, 2.0, and 2.1, respectively\(^2\).

### 5.2.1. Museme a

Compared with its hypothesised source from Bach, Museme a’ in Beethoven is only slightly changed rhythmically. In terms of Cope’s categories, it is a Commonality, on account of its being relatively short and consisting of a compound triadic \((g^\#-e,^\#(c^2))-scalic (5–2)\) pattern. It also has sufficient salience to register as a weak Quotation, despite the rhythmic and metric differences between the two versions (see Section 3.6, Category (ii)). As Figure 12 shows, the difference between Bach’s and Beethoven’s form inheres in the temporal, not the spatial, aspects of the spatiotemporal firing pattern (S[TFP]): the same basins of attraction are implicated, and only the intervals between their firings differ. This distinction offers a means of understanding the difference between pitch and rhythmic musemes. The former are in some sense more fundamental, constituting the outline upon which the latter are hung. Pitch musemes are implemented, as real entities, by firing neurons, whereas rhythmic musemes are more virtual, being the inter-onset (IOI) and/or offset-to-onset (OOI) intervals between those firings (Temperley 2001: 27, 68). If a rhythmic museme is taken (as this view implies) to be a separate entity from a pitch museme, then Museme \(a/a’\) in Bach and Beethoven is the same pitch museme associated with two different rhythmic musemes. Conversely, if some degree of pitch-rhythm blending is taken to occur (if, in other words, this is an S+TFP, not an SFP and a TFP), then Beethoven’s museme is a variant of Bach’s.
Example 7. Summary of musemes in Example 6

Museme $a$

\{ \{75,11\}, \{74,12\}, \{79,13\}, \{77,15\}, \{75,17\}, \{74,19\} \}

Museme $a1$

\{ \{75,9\}, \{74,11\}, \{79,13\}, \{77,15\}, \{75,17\}, \{74,19\} \}

Museme $b$

\{ \{72,11\}, \{71,12\}, \{69,13\}, \{67,14\}, \{65,15\}, \{64,16\} \}

Museme $b1$

\{ \{72,1\}, \{70,3\}, \{68,5\}, \{67,7\}, \{65,9\}, \{63,11\}, \{68,13\} \}

Museme $c$

\{ \{74,1\}, \{72,15\}, \{74,16\}, \{72,17\} \}

Museme $c1$

\{ \{74,9\}, \{72,23\}, \{74,24\}, \{72,25\} \}
Example 7 (cont.). Summary of musemes in Example 6

Example a)

Museme A

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}

Museme A1

\{ 74, 9, 80, 10, 77, 11, 74, 12, 71, 13, 77, 14, 74, 15, 71, 16, 68, 17, 74, 18, 71, 19, 68, 20 \}

Example b)

Museme B

\{ 60, 1, 65, 9, 58, 17, 63, 25, 56, 33 \}

Museme B1

\{ 72, 9, 65, 45, 70, 81, 75, 105, 68, 169 \}

Example c)

Museme C

\{ 55, 16, 59, 17, 62, 21, 65, 25 \}

Museme C1

\{ 55, 1, 59, 9, 62, 21, 65, 25 \}

Example d)

Museme D

\{ 74, 1, 72, 5, 60, 1 \}

Museme D1

\{ 74, 9, 72, 21, 60, 9 \}

Example e)

Museme E

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}

Museme E1

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}

Example f)

Museme F

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}

Museme F1

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}

Example g)

Museme G

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}

Museme G1

\{ 80, 2, 77, 3, 80, 4, 74, 5, 77, 6, 71, 7, 74, 8, 68, 9 \}
Figure 12. Hypothesised cortical encoding of Museme a

Table 2. Earth mover’s distance for Museme a according to five metrics

<table>
<thead>
<tr>
<th>Museme n Museme Elements</th>
<th>(i) Rhythmic Weight</th>
<th>(ii) Krumhansl Key-Profile</th>
<th>(iii) Temperley I Key-Profile</th>
<th>(iv) Temperley Line-of-Fifths</th>
<th>(v) Jan I-Rp</th>
<th>(vi) Average of (i)–(v)</th>
<th>(vii) Normalised (Average/n Museme Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a → a'</td>
<td>0.2</td>
<td>0.547546</td>
<td>0.510204</td>
<td>0.4375</td>
<td>0.496933</td>
<td>0.438437</td>
<td>0.07307</td>
</tr>
<tr>
<td>6 → 6</td>
<td></td>
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</table>

The EMD values for Museme a are the lowest of the seven musemes, as might be expected for a ‘Quotation-Commonality’ of this type: there is not a great deal of earth to move here, and it is not shifted a substantial distance, so the effect of the weightings is not significantly amplified. The Krumhansl and Temperley weightings give the highest values, perhaps on account of the rhythmic relocation of the relatively heavily weighted 3 and 5. While a high i-rp value generally indicates a significant cognitive surprise, such as might occur when a Registral Process (Narmour 1990: 330–1) subverts an expected Process, a high i-rp-EMD might also result from the rectification of a surprising pitch sequence in the source museme (such as a Registral Process) back to one which is more cognitively normative in the copy (such as a Process). In such a situation, a high i-rp-EMD might correlate inversely with perceptual-cognitive salience.
Figure 13. Hypothesised cortical encoding of Museme b

Table 3. Earth mover’s distance for Museme b according to five metrics

<table>
<thead>
<tr>
<th>Museme Elements</th>
<th>(i) Rhythmic Weight</th>
<th>(ii) Krumhansl Key-Profile</th>
<th>(iii) Temperley I Key-Profile</th>
<th>(iv) Temperley Line-of-Fifths</th>
<th>(v) Jan I-Rp</th>
<th>(vi) Average of (i)–(v)</th>
<th>(vii) Normalised (Average/n Museme Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b → b'</td>
<td>3.10903</td>
<td>5.91749</td>
<td>6.29256</td>
<td>6.82161</td>
<td>5.50881</td>
<td>5.5299</td>
<td>0.850754</td>
</tr>
<tr>
<td>6 → 7 (n = 6.5)</td>
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5.2.2. Museme b

Museme b' in Beethoven undergoes a mode-shift from its posited Haydn source plus the addition of an extra pitch. As with Museme a, this museme might be regarded as a Quotation-Commonality intersection (Section 3.6, Category (ii)): analogously, it is a small enough fragment to be regarded as a commonplace figure in eighteenth- and early-nineteenth-century style, yet its similarity to the posited Haydn source, particularly given the broader contextual affinities discussed in Section 5.1, suggests some level of awareness on Beethoven’s part of the earlier work[24].

The EMD values for Museme b are, perhaps unsurprisingly, higher than those for Museme a: not only is there the effect of the rhythmic and mode shifts (three of Haydn’s pitches are changed by a semitone in Beethoven), but Beethoven’s addition of a terminal minim (half note) means a hole deeper than any of the others must be excavated and filled with earth from the other pitches. Again the Krumhansl and Temperley values are high here (but not higher than the line-of-fifths, owing to sometimes jerky motion across the line arising from...
the effect of the mode-shift), with Temperley giving the higher of the two values perhaps on account of his greater weighting of the shift from 7 to 7 than is the case in Krumhansl. It might be argued that mode-shift EMD calculations (here and in the case of Musemes c, f, and g) should be based on intra-weighting-set values (i.e., major to minor shifts should use major values only and vice versa); but here inter-weighting-set values (i.e., major to minor shifts use major and minor values respectively) are used. This is on the grounds that it is difficult to verify that a specific minor-key copy is perceived in any given transmission situation as a variant of a specific major-key source.

5.2.3. Museme c
As one of the most ubiquitous patterns in late-eighteenth-century style, the trill of Museme c is a Commonality. It is represented in Example 7 as a four-note museme (initial note, terminal note, plus the two-note connecting Nachschlag), although of course, in reality, it consists of an unmeasured and rapid alternation between these two pitches (see Figure 13 and Table 3).

By treating Museme c as a four-note pattern, as opposed to a multi-note oscillation, an inappropriately high EMD is avoided. Nevertheless, while the EMD values for Museme c are quite consistent across metrics here, they are also still high. This may be the result of the copy form’s starting on the third beat of the bar and having a terminal minim, not a crotchet (quarter note) as in the source. While this might, on one level, reflect a significant conceptual difference between the two forms (a 4/4 unsyncopated version is mutated to a 3/4 syncopated version), the perceptual difference between them is arguably not as great as these EMD values might imply. Such metrical asynchrony with respect to the source naturally increases the distance across which the earth needs to be moved. One could argue that normalisation to a common starting beat (which might also occur in perception when musemes are mapped against LTM-stored sources) might produce more meaningful EMD data. Nevertheless, syncopations of this type contribute significantly to the perceptual-cognitive salience of musemes and it therefore seems desirable to incorporate them when the EMD is being utilised as an index of this attribute, even at the risk of mismatch between the conceptual and perceptual dimensions.

5.2.4. Museme d
Museme d might be regarded as a Likeness-Commonality intersection (Section 3.6, Category (v)), in that Beethoven’s copy form bears a general similarity to Mozart’s source, but the resemblance is perhaps best understood as the result of the operation of a musicoperational/procedural meme upon a commonplace stylistic particle. That is, composers of the late-eighteenth century were aware that a particularly brilliant effect could be achieved by arpeggiating a diminished seventh chord over a pedal, resulting in a dominant minor ninth harmony. Whether Beethoven was specifically influenced by the passage from Mozart’s K. 466 (an interpretation given weight by their shared tonal context and Beethoven’s familiarity with the work (see Endnote 41)) is difficult to determine. But it seems likely that if the passage from K. 466 was not the sole, unmediated source for that from op. 106, then it may well have constituted a member of the set from which Beethoven generated his Museme d and/or extrapolated his ‘Meme x-d’ – the ‘arpeggiate a diminished seventh’ musicoperational/procedural meme operating upon Museme d (see Figure 15 and Table 5).

While EMD values are naturally contingent upon museme segmentation and the resulting pitch content, the segmentation of Museme d is more problematic than for the other musemes considered here on account of the somewhat abstract, musicoperational/procedural nature of these patterns. The octave/eight-pitch range chosen on conceptual (abstract harmonic) grounds as the segmentation criterion of the Mozart form maps onto the octave/twelve-pitch range chosen on gestalt, harmonic, and conceptual grounds for the Beethoven. The four extra pitches in Beethoven’s museme d increase the amount of earth moving, as does the lack of alignment between the cycles through which the diminished-seventh arpeggio is rotated in the two forms.

5.2.5. Museme e
Museme e is probably one of the most durable of all musemes in the common-practice period. This survival is perhaps related to the fact that it almost invariably occurs as a middleground-level Framework generated by repeated foreground-level patterning. In most Baroque- and Classical-period contexts, the foreground museme is either a generic arpeggio figure or a short and distinctive motive, both often Commonalities. The cycle itself is more abstract, and might be represented either as a series of chords (as perhaps best suits Bach’s instantiation) or as a series of monadic pitches (as perhaps best suits Beethoven’s). As a foreground-middleground musemeplex, the middleground museme exists in a symbiotic (or ‘sym-musemic’) relationship with the foreground museme(s). Both benefit from this
The EMD values for Museme $c$ are, by an order of magnitude, far higher than those for the other musemes, on account of the fact that Bach’s localised and short-term progression is vastly expanded in Beethoven. Bach’s small heaps of earth are spread over a wide terrain in Beethoven’s version. The discrepancy presented by the rhythmic weight value, the most significant in the whole data-set, can be explained by the fact that the pitches of the copy museme have a much greater weight with respect to those of the source than is the case with the other systems, so energy does not need to be expended in the work of spreading out a similar weight very thinly. Given what was said above about the representation of the cycle of fifths in these passages, greater care is needed here than is the case with the other musemes in applying the EMD to what might be regarded as somewhat intangible phenomena, which are reified in this instance (as chords
**Figure 15.** Hypothesised cortical encoding of Museme $d$

![Cortical Encoding Diagram](image)

**Table 5.** Earth mover's distance for Museme $d$ according to five metrics

<table>
<thead>
<tr>
<th>Museme Elements</th>
<th>(i) Rhythmic Weight</th>
<th>(ii) Krumhansl Key-Profile</th>
<th>(iii) Temperley I Key-Profile</th>
<th>(iv) Temperley Line-of-Fifths</th>
<th>(v) Jan I-Rp</th>
<th>(vi) Average of (i)–(v)</th>
<th>(vii) Normalised (Average/n Museme Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d \rightarrow d'$</td>
<td>7.1693</td>
<td>7.3202</td>
<td>7.17764</td>
<td>7.13371</td>
<td>7.28908</td>
<td>7.21799</td>
<td>0.721799</td>
</tr>
<tr>
<td>$8 \rightarrow 12$ (n = 10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

or as single pitches) for the purposes of measurement and comparison. Such high EMD values would suggest that no relationship exists between these passages, but perceptually and conceptually it is clear that the Bach and Beethoven passages are related, however obliquely.

### 5.2.6. Museme $f$

A dominant-seventh arpeggio is one of the most fundamental Commonalities in tonal music. Mozart's version is rendered highly salient by virtue of its rhythmic presentation, with the semiquaver upbeat and interstitial quaver rests imparting to it a markedness not normally found in other occurrences of this allele-class. Given its temporal extension, Beethoven's form is more a Framework-Commonality intersection (Section 3.6, Category (vi)), the Commonality being a 'superstructure', to use Cope's term, for the trills (Museme $c$) which increasingly dominate the texture by this point (see Figure 17 and Table 7).
Figure 16. Hypothesised cortical encoding of Museme  

Table 6. Earth mover’s distance for Museme  

<table>
<thead>
<tr>
<th>Museme Elements</th>
<th>(i) Rhythmic Weight</th>
<th>(ii) Krumhansl Key-Profile</th>
<th>(iii) Temperley I Key-Profile</th>
<th>(iv) Temperley Line-of-Fifths</th>
<th>(v) Jan I-Rp</th>
<th>(vi) Average of (i)–(v)</th>
<th>(vii) Normalised (Average/n Museme Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e → e'</td>
<td>14.1809</td>
<td>62.6543</td>
<td>62.4085</td>
<td>66.537</td>
<td>63.9391</td>
<td>53.944</td>
<td>10.788792</td>
</tr>
<tr>
<td>5 → 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The EMD values for Museme  reflect the fact that while the pitches of the source form are not changed in the copy, their rhythmic position is altered. As with Museme  this metrical asynchrony between the two forms augments the EMD values compared with that of a hypothetical copy which also began on an upbeat. One aspect of salience not captured by these values, even the Krumhansl/Temperley ones, is that which results from top-down processing  . While this holds for other musemes considered here, the dominant seventh chord outlined by Museme  has particular cultural-stylistic implications in Western tonality, in that it engenders a sense of tension and instability. This attribute interacts with the individual tonal stability weightings of its component pitches and the innate, bottom-up factors captured by the i-rp. Thus, and as with most musemes, there are several competing forces at play here: the innately relatively stable 4 and 5 degrees of this museme combine in a way which creates culturally mediated instability; yet the relatively high
EMD values motivated by these scale degrees align with the high perceptual-cognitive salience which arguably results from the instability. Another aspect of salience not captured here is a consequence of the interstitial rests in Mozart’s museme, which afford it a greater clarity and pointedness than a straight-crotchet form. My utilising IOI and not OOI means this attribute does not factor into the EMD.

5.2.7. Museme $g$

Museme $g$, a cadential Commonality, raises the issue of the minimum number of pitches required to constitute a museme. It might be maintained that three elements is the minimum lower threshold, because a two-element pattern in isolation appears to lack the necessary information density to function as a viable replicator (Jan 2007: 61). But Museme $g$ arguably cannot be understood simply as a two-element pattern and must instead be seen in its wider context, as a bi-parametric (pitch-rhythm) element of a multiparametric (pitch-rhythm-harmony) style-structural cadence. On account of this affordance of museme status by its surrounding context, it is accepted here as a two-element melodic museme. For the purposes of calculating its i-rp, it is understood as an incomplete ($\{x\}$-y) Process, where $x$ is assumed to be two semitones on the basis of backward, reductive extrapolation to (in the case of Mozart’s museme $g$) the previous $c^\flat$ of b. 72 and (in the case of Beethoven’s museme $g'$) an implicit preceding $a^\flat$ imagined in b. 119 (see Figure 18 and Table 8).

As with Museme $c$, the EMD values for Museme $g$ are relatively high on account (certainly in the Krumhansl and Temperley metrics) of the tonic-degree-heavy source being rhythmically relocated in the copy (again, metrical asynchrony occurs here). The high values may also be an artefact of the calculation methodology, which requires the algorithm to consider the lower voice as linearly succeeding the second upper-voice pitch and not, as occurs in reality, as accompanying the upper line. Similarly to Museme $c$, the difference between the two forms in perception is arguably not as great as the EMD values might imply: they are probably perceived as members of the same allele-class, and their common structural function attenuates the metrical differences between them. With both of these musemes, the relatively high EMD values therefore appear to align more closely with conceptual, as opposed to perceptual, differences.

5.2.8. Summary

The EMD values for Musemes $a$–$d$, $f$, $g$ are graphed on Figure 19 (i) and Museme $e$ is given a separate graph, Figure 19 (ii), on account of its very high EMD values, giving a visual impression of the distance between the seven musemes’ sources and copies in EMD terms.

To summarise the often very disparate data presented here, it is clear that Cope’s categories relate to the EMD in complex ways. Despite the length necessary for recognition as such, Quotation will tend to have a low EMD because there is, by definition, little movement between source and copy, unless the Quotation is associated with consistent rhythmic augmentation or diminution, or if transposition is taken into account. While ‘pure’ (unintersected) Quotation is not a significant feature of the music considered in Section 5, this point is supported to some extent by Musemes $a$ and $b$ (Quotation-Commonality intersections). Similarly Commonalities, by virtue of their brevity and resistance to mutational change, will tend to have low EMDs, unless employed as Frameworks (Museme $f$), or unless distorted by metrical asynchrony (Musemes $c$ and $g$). The middle three categories might be expected to motivate the highest EMDs, given that relating examples of Paraphrase (which does not occur here), Likeness, and Framework often involves significant manipulations in both pitch and rhythm, but this is only true here in the case of Museme $e$ and not Museme $d$.

Contrary to Figure 11, it is clear that conceptual or perceptual similarity between musemes (where a relationship can be clearly understood or heard, respectively, despite dissimilarity of notation) does not always correlate with low EMD values, especially when metrical asynchrony is the cause of the notational dissimilarity. Such inappropriately high EMD values are thus often the result of using notation, as opposed to sound, as the source of the input for EMD calculations. Moreover, assessments of a copy museme’s perceptual-cognitive salience vis-à-vis its source arrived at through subjective introspection may not necessarily correlate with the salience implied by its EMD values, even allowing for the difficulties of comparing a numerical value with a psychological response and also allowing for divergences between subjects’ assessments. Nevertheless, the data considered here suggest that the EMD offers a consistent, stable, and objective index of the relationships between a set of musemes in terms of the attributes of similarity and salience. The correlation of these attributes with listeners’ assessments appears readily amenable to empirical testing using EMD values as a frame of reference.
**Figure 17.** Hypothesised cortical encoding of Museme $f$

![Hypothetical cortical encoding diagram](image)

**Table 7.** Earth mover’s distance for Museme $f$ according to five metrics

<table>
<thead>
<tr>
<th>Museme Elements</th>
<th>(i) Rhythmic Weight</th>
<th>(ii) Krumhansl Key-Profile</th>
<th>(iii) Temperley I Key-Profile</th>
<th>(iv) Temperley Line-of-Fifths</th>
<th>(v) Jan I-Rp</th>
<th>(vi) Average of (i)–(v)</th>
<th>(vii) Normalised (Average/n Museme Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f \rightarrow f$</td>
<td>2.58032</td>
<td>6.46814</td>
<td>6.21875</td>
<td>4.27273</td>
<td>5.79021</td>
<td>5.06603</td>
<td>1.266508</td>
</tr>
<tr>
<td>$4 \rightarrow 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 18. Hypothesised cortical encoding of Museme $g$

Table 8. Earth mover’s distance for Museme $g$ according to five metrics

<table>
<thead>
<tr>
<th>Museme $n$ Museme Elements</th>
<th>(i) Rhythmic Weight</th>
<th>(ii) Krumhansl Key-Profile</th>
<th>(iii) Temperley I Key-Profile</th>
<th>(iv) Temperley Line-of-Fifths</th>
<th>(v) Jan I-Rp</th>
<th>(vi) Average of (i)–(v)</th>
<th>(vii) Normalised (Average/$n$ Museme Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g \rightarrow g'$</td>
<td>7.41202</td>
<td>11.1301</td>
<td>10.963</td>
<td>12.0</td>
<td>10.6667</td>
<td><strong>10.43436</strong></td>
<td><strong>3.478121</strong></td>
</tr>
<tr>
<td>$3 \rightarrow 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 19 (i). Values of EMD for Musemes a–d, f, g

Figure 19 (ii). Values of EMD for Museme e
6. Conclusion: The EMD and Population Memetics

In a somewhat speculative conclusion, I will briefly consider some of the implications of the museme categories adapted from Cope, and the associated EMD calculations, to make some predictions concerning what they might tell us about the inhabitants of the wider museme-pool.

Imagine the possibility of a computer program — one might call it Aristoxenus, after the first known Western music theorist-taxonomist — which could scrutinise every piece of music in a given corpus, segment it into cognitively meaningful units at various structural-hierarchic levels (Lerdahl 1992), and then cross-map these units chronologically. This would allow one to build up a near-complete picture — possibly in the form of a vast multidimensional musemic hypervolume (Section 4.3) — of all the musemes in that corpus and the nexus of transmission which links them. Of course, while there are programs which can do some of these things (Lartillot 2009; Hawkett 2013), albeit not always with a specifically memetic focus, and others which can simulate the processes which might be taking place within such museme-pools (Miranda et al. 2003; Gimenes and Miranda 2011; MacCallum et al. 2012), none to my knowledge can accomplish the complete task, namely to offer a comprehensive description of a given museme-pool and to chart its population memetics in detail.

But if this possibility were to become a reality, what might the issues discussed here allow us to hypothesise as to the nature of the revealed museme-pool? Firstly, it may be the case that a given museme’s representation in the museme-pool will be in direct proportion to its perceptual-cognitive salience, however this is quantified. If the intra-museme EMD (Section 4.4) is taken as one measure of this attribute, then a given museme’s representation in the museme-pool will be proportional, all other things being equal, to this value (as hypothesised in Figure 11, although this correlation will presumably not continue indefinitely). Naturally this is not universally true, and there will always be exceptions owing to myriad other factors which impinge upon museme replication. But given that there are many millions of musemes involved in countless replication processes, there should be a general correlation between how perceptually-cognitively salient each museme is and how many copies of it exist in the museme-pool. In this respect, the EMD is effectively a measure of museme ‘selfishness’ (Dawkins 1989: 11).

Example 8 illustrates this with two hypothetical musemes, each beginning with the ‘Jupiter’ schema (Gjerdingen 2007: 116–7), but ending differently. Using Temperley’s key-profile weightings, Example 8 (i) has an intra-museme EMD of 4.89796 and Example 8 (ii) an intra-museme EMD of 5.11364. This difference perhaps reflects the instability of the modulation to the dominant key in Example 8 (i), and the higher tessitura this particular implementation of the modulation involved. While impossible to verify without the aid of Aristoxenus, the ‘correlation hypothesis’ suggests that the copy form will be more numerous in the museme-pool than the source.

Secondly, the survival of a mutant museme will relate, again, to the EMD. A low inter-museme EMD (Section 4.4) means that the copy form is not significantly different to its source, and may still occupy the same museme allele-class. A high inter-museme EMD, by contrast, may be a marker of the copy form’s leaving the parent allele-class and either joining another, extant, allele-class, or initiating a replicative process which leads to the formation of a new allele-class (Jan [n.d.]). Which of these scenarios is more advantageous is difficult to generalise upon. A well-represented allele class will afford a relatively (proto)typical (Gjerdingen 1988: 94, 103–4) variant-copy a replicative ‘umbrella’, whereas a less (proto)typical variant might outstrip its parent allele-class in terms of replicative vigour (‘fecundity’, to use Dawkins’ term (Dawkins 1999: 194)). The least (proto)typical variants, while they may potentially be replicative high-flyers, run the risk of falling, Icarus-like, to earth (unless they can be ‘caught’ by a second, conformant, allele-class), on account of their radical deviations from established allele-class norms.

Finally, the EMD acts as a kind of evolutionary brake upon excessive mutation. More precisely, it serves as a predictor of which museme variants are likely to prove too difficult for listeners to comprehend and too singular to fit (conceptually) with other musemes in the pool or (juxtapositionally) in real musical contexts. Such musemes push the development of what is often termed musical style, but they strain against the forces of conservatism. The latter forces stem from the innate perceptual-cognitive capacities of human beings, from the inherent inertia of the museme-pool, and from the verbal-conceptual memes which regulate what is considered tasteful and acceptable in music. But mutant musemes eventually prevail, for musemic evolution rarely stands still when the wider memetic environment is
Example 8 (i). Intra-museme EMD: source museme

\[ \{\{60,1\}\}, \{62,5\}, \{65,9\}, \{64,13\}, \{62,17\}, \{59,21\}, \{60,25\}\]  

Example 8 (ii). Intra-museme EMD: copy museme

\[ \{\{60,1\}\}, \{62,5\}, \{65,9\}, \{64,13\}, \{62,17\}, \{66,21\}, \{67,25\}\]  

dynamic; and by doing so they move the centre of musemic gravity towards themselves — they remake the musemic world in their own image — with consequences which, at their most extreme, include the reconfiguration of higher-order musical systems such as those regulating tonal organisation.

ENDNOTES

1. Some of the sources cited in this article are from the two *Musicae Scientiae* discussion fora on musical similarity (Various 2007, 2009) and two volumes of *Computing in Musicology*, on melodic similarity and music query (Hewlett and Selfridge-Field 1998, 2004). These four volumes, together with the proceedings from the ongoing ISMIR/MIREX conferences (<http://www.ismir.net/>; <http://dc.ofai.at/browser?>; <http://www.music-ir.org/mirex/wiki/MIREX_HOME>), offer overviews of the subject from a variety of perspectives.

2. Ideally, but not always in practice, psychological and music-theoretical/analytical perspectives on similarity (and other aspects of music) will align (Gjerdingen 1999).

3. Indeed, Nattiez argues that there is no consensus in the theoretical and analytical literature — rather, there is a Tower of Babel — when it comes to the definitions of such common music-theoretical units as cells, motifs, themes, and phrases (Nattiez 1990: 156–60).

4. One must nevertheless bear in mind that the homologous transmission of memes might introduce significant differences, whereas analogies might arise through accidentally similar circumstances. This is reminiscent of the situation in zoology where (for instance) octopus and human eyes are very similar but are not homologous; whereas arms and wings are very different, but are homologous. I am grateful to Alan Marsden for this point.


6. A memotypic form of a museme is its encoding within the brain; the phemotypic form is its manifestation in the world — as part of artefacts (such as musical scores, CDs, or sound waves) or behaviours (such as playing and singing).

7. I use Tagg’s term, ‘museme’ (Tagg 1999: 32), in a somewhat different sense to his, as a neological contraction of ‘musical meme’.

8. The attributes on the X axis of Figure 1 are to be understood as follows. ‘Potential copying-fidelity’ (after Dawkins’ ‘copying-fidelity’ (Dawkins 1989: 194–5)) refers to the likelihood of a museme being miscopied. ‘Perceptual-cognitive salience’ relates to the degree to which a museme is felt to be in some sense striking. ‘LT Memorability/ST Non-Memorability’ relate to the fact that a low-element-number museme may be relatively difficult to store in Long-Term Memory (LTM), but relatively easy to retain in Short-Term Memory (STM), and vice versa. The longer a museme the more likely it is to consist of the distinctive collection of generic ‘style shapes’ (features) which characterises a ‘style structure’ (schema). The ‘information content’ of a museme is simply the network of relationships between its elements, which naturally increases with the number of museme-elements according to \( r = n \times (n-1) \), where \( r \) is the number of intra-museme relationships and \( n \) is the number of museme elements. The longer a museme, the greater the likelihood that it will be regarded as deriving (by quotation, borrowing, or plagiarism) from a specific antecedent work. As noted in Section 1, ‘analogy’ and ‘homology’ relate to fortuitous and evolutionarily motivated resemblances between musemes respectively, likelihood of the former decreasing and of the latter increasing as the number of museme elements increases (and vice versa). The remaining features of Figure 1, relating to Cope’s ‘continuum’, are discussed below.
9. It should be stressed that many attributes other than length affect the salience of a museme. Some of these are considered here, but a comprehensive treatment is beyond the scope of this article.

10. In this sense, phrases are sometimes, but not invariably, equivalent to musemes.

11. Of course, emulating a given composer's style requires more than the mere replication of characteristic patterning. For one thing, the sequential ordering of material is arguably as important as that which is ordered; but the starting point, certainly in computer implementations, is the raw material itself.

12. Such allusions may, of course, also be identified 'manually' by a human observer, as I do in Section 3, albeit sometimes using the ThemeFinder website (Huron et al. 2014) to test introspective intuitions about pattern similarity. The latter is a web interface, implemented by Andreas Kornstädt and Craig Sapp, to the theme incipit-searching tool from David Huron’s Humdrum Toolkit (Huron 1994; Kornstädt 1998). The Humdrum patt and pattern tools offer further functionality for pattern searching in large databases of Humdrum-format **‘kern’** scores (Huron 2002; Jan 2004; CCARH 2014).

13. While Langrish may be accused by some of unnecessary multiplication of neologisms, his terms are quite useful in connection with what I call 'musico-operational/procedural memes'. As Langrish argues, 'Recipemes are competing ideas of how to do things', 'selectemes are competing ideas of betterness', and 'explanemes are competing ideas that are used in answering questions about why things work or work better' (Langrish 1999: Secs. 4.2, 4.3; see also Jan 2011: Sec. 2). Such abstract ideas — what might be termed 'verbal-conceptual memes' (Jan 2007: 18) — are quite different from what I, after Langrish, call 'unitemes', examples of which are the concrete and discrete musical patterns with which this article is primarily concerned.

14. This is also an index of a pattern's population (Gjerdingen 1988: 99–105).

15. This is also an index of a pattern's perceptual-cognitive salience (Jan 2007: 62, Figure 3.2), and of the probability of a relationship of homology as against analogy between 'source' and 'target' patterns.

16. It could be argued that musemes must consist of at least two (harmonic) or three (melodic) elements (see Section 5.2.7).

17. If the same set of musemes is common to two phrases, then one may speak of the replication of a *musemeplex*: a replicated complex of musemes, the individual musemes of which are also replicated independently in other contexts (the latter stipulation of course affording them the status of musemes) (Jan 2007: 83–94).


19. Cope is presumably referring to 'strong' replication between composers, not self-quotation — such as Haydn's apparent quotation of the opening of his Farewell Symphony, no. 45 (1772), at b. 109 of Symphony no. 60, I (1774), and at b. 62 of Symphony no. 85, I (1785).

20. This concept refers to an evolutionary relationship between, for instance, a melodic and a harmonic museme, such that they are often replicated together despite remaining ostensibly separate patterns.

21. It could be argued there are five notes in common here, the passing-note d½ (decorated in Mozart) connecting e½ to a terminal c½ in both cases. The criteria for determining similarity between the harmonies of two passages — which can intensify or attenuate the perceived similarity between melodic and/or rhythmic musemes — are beyond the scope of this article.

22. To paraphrase Gjerdingen (1988: 102), 'the constituent [musemes] in a [musemeplex] should all have broader historical distributions and less pointed population curves than the [musemeplex] itself, because the definition of the [musemeplex] stipulates additional constraints of context and interrelationship'.

23. See Grove (1896: 222–3) for another possible folk song Quotation in the finale of this symphony.

24. A detailed musemic breakdown is unnecessary here, but suffice to say that Bach's foreground-level triplets are replaced by tremolo, and the melodic figures are subtly altered, sometimes by the addition of the galant upbeat turn figure.

25. Temperley gives the date of this recording as 1918, but this must be incorrect because Joplin died in 1917.

26. These include the resemblance between a figure from Liszt's *Ich möchte hingeihn* and other works and the opening of Wagner's *Tristan und Isolde* in his Figure 6 (Cope 2003: 15; see also Jan 2007: 150).

27. Given how little is known of medieval performance practice, caution is needed when using the spacings and beamings of modern notation (spacings in Examples 3 (i) and 3 (ii) simply replicate those employed to facilitate alignment with other, longer, examples in Treitler's Example 4) to represent what might have been temporally equidistant pitches in performance. In such situations, coindexation-determined segmentation, changes of melodic direction, and the structure of the text have to compensate for the lack of duration-based grouping cues in demarcating these tropes' component musemes. I am grateful to Lisa Colton for helpful discussions of this trope.

28. See Jan (2007: 49–52) for details of the museme-analytical symbology utilised in this article.

29. Borrowed from genetics, the concept of museme alleles refers to two musemes with a common sub-foreground (schematic) structure and analogous function, despite ostensible surface-level differences (Jan 2010: 11–15).
30. Strictly, it is a 1–7…4–3 schema (Gjerdingen 1988: 64), because his later conception of the Meyer (Gjerdingen 2007: 111–12; 459) stipulates a ①②③④① bass line, whereas Gjerdingen’s earlier category permitted a number of different bass motions, including the ①②③④ occurring in the Haydn phrase.

31. I am grateful to Trevor Rawbone for drawing my attention to the revigoration of galant schemata in such World War II popular music. Note that the following four bars of Lili Marleen arguably instantiate a ‘Prinner’ (①②③④①②③④) schema (Gjerdingen 2007: 46, 455).

32. That is, a group of discrete entities occupying multidimensional space, such as the notes of a melody, each assigned a relative weighting.

33. Clearly the same is true for analogically related replicators, but in this case the proximity is fortuitous and not a consequence of the operation of evolutionary processes.

34. Donald O. Hebb (1949) argued in the 1940s that interconnected groups of neurons functioned as ‘the unit of perception — and therefore memory’ (Calvin 1996: 13, 104).

35. In this sense, intra-museme EMD has its deficiencies as a cognitive model.

36. The grid upon which the triangular arrays are placed, wherein raised circles represent minicolumns, is taken from <http://williamcalvin.com/Demo2.htm> (accessed 2 October 2014).

37. Here there is no rhythmic change, only a change of pitch. In the diagram of Section 5.2, sometimes the opposite is the case (i.e., rhythmic change without a change of pitch). In this situation, the presence of a dotted triangle without a glowing red circle signifies rhythmic and/or metrical change but not pitch change.

38. EMD values here and elsewhere in this article are calculated using an implementation based on Yossi Rubner’s C code for this equation (Rubner 1998). I am grateful to Jonathan Wakefield for his assistance with this implementation. While Typke and his colleagues use additional segmentation and translation processes to compute overall melodic similarity, only the original EMD (Rubner) algorithm is utilised here.

39. One reason the passage was chosen was because of the way in which these antecedents, which are relatively commonplace, are often radically mutated to create their consequent forms.

40. The second is the introduction of subject 2 in retrograde at b. 153.; the third is the introduction of subject 3 at b. 250.

41. According to Thayer, Beethoven performed this concerto on 31 March 1795 at a charity concert at the Burgtheater organised by Constanze Mozart (Thayer and Forbes 1967: 17) and he wrote cadenzas (WoO 58) for the first and third movements around 1809. Block argues for the influence of Mozart’s K. 466 and K. 491 (1786) on Beethoven’s first two piano concertos (op. 15, 1800; op. 19, 1798) (Block 1991; see also Kramer 1991).

42. The total weight of each point-set (museme) under comparison is allowed to be different, so partial matching is possible.

43. In Krumhansl’s and Kessler’s key-profile (which forms the basis of the Krumhansl-Schmuckler (KS) algorithm (Krumhansl 1990)), the major-mode weightings are: ① = 6.35, ① = 2.23, ② = 3.48, ③ = 2.33, ④ = 4.38, ⑤ = 4.09, ⑥ = 2.52, ⑤ = 5.19, ③ = 2.39, ⑥ = 3.66, ④ = 2.29, ⑦ = 2.88; and the minor-mode weightings are: ① = 6.33, ① = 2.68, ② = 3.52, ③ = 5.38, ④ = 3.83, ⑤ = 6.40, ③ = 3.54, ④ = 2.54, ⑤ = 4.75, ④ = 3.98, ⑤ = 2.69, ③ = 3.34, ⑦ = 3.17. These are what Temperley terms ‘Neutral Pitch Class’ (NPC) values, not ‘Tonal Pitch Class’ (TPC) values. That is, they do not take account of note spelling in specific contexts (Temperley 2001: 118, 183). See Sapp (2011: 95) for a comparison of KS with other key-finding algorithms.

44. In Temperley’s NPC key-profile (specifically that which he terms ‘Temperley I’, to distinguish it from a variant, ‘Temperley II’), the major-mode weightings are: ① = 5.0, ① = 2.0, ② = 3.5, ③ = 2.0, ④ = 4.5, ⑤ = 4.0, ⑥ = 2.0, ⑤ = 4.5, ③ = 3.5, ⑥ = 2.5, ⑤ = 1.5, ⑦ = 4.0; and the minor-mode weightings are: ① = 5.0, ① = 2.0, ② = 3.5, ③ = 4.5, ④ = 2.0, ⑤ = 4.0, ⑥ = 2.0, ④ = 2.0, ⑤ = 4.5, ⑥ = 3.5, ③ = 2.0, ⑦ = 1.5, ⑥ = 4.0 (Temperley 2001: 180–1). It is only the ‘mode-defining’ degrees (3 and 6; shown in bold) which differ between the two systems, the diatonic values for the major-mode degrees (③, ⑥) becoming the diatonic values for the corresponding minor-mode degrees (④, ⑤), and vice versa for the non-diatonic values.

45. The line is as follows, wherein each consecutive movement counts as 1.0 and the initial node of the museme, whichever pitch this is, is given a notional value of 1.0: F=①–②–E⑤–A④–D③–G⑩–C⑨–F⑧–B⑧–E⑦–A⑥–D⑤–G⑧–C⑨–F⑧–B⑧–E⑦. In this sense, the line-of-fifths value is a kind of intra-museme EMD.

46. Such attributes could also have been quantified as additional dimensions in the ground-distance space rather than weights. Having musical features as weights raises the issue, not addressed here, of how to balance the influence of weight versus ground distance. Certainly, when a weight is the (additive, multiplicative) combination of two or more orthogonal musical features (such as pitch and rhythm), that information is necessarily collapsed to a single dimension of weight.

47. See Jan (2007: 129–33, Table 4.1), for suggested i-rp values for Narmour’s (1990, 1992) implication-realisation structures. See also Schellenberg (1996, 1997).

48. These symbols mean ‘Retrospective Registral Reversal’ (Narmour 1990: 335), ‘Retrospective Reversal’ (Narmour 1990: 259–61), and ‘Process’, respectively.
49. In the analyses, IC6, the augmented fourth/diminished fifth, is treated according to Narmour (1990:79). That is, it may be taken to imply either Process or Reversal, according to context.

50. As suggested in Section 2, whether this connection is the result of a conscious decision by Beethoven to adapt Haydn’s music, or whether it is the result of a subconscious echo of the past, is immaterial from the memes’ eye view. But the greater the degree of consciousness or intentionality on Beethoven’s part, however this is ascertained and quantified, the more strongly we can regard the affinity as a Quotation.

51. See, for instance, bb. 22—262 of the C minor Fugue from Book II of Bach’s Das wohltemperirte Clavier, where a complete revolution around the cycle is articulated by the distinctive opening rhythm of the subject. By the next entry of the subject, b. 263, the cycle has generated a satisfying amount of musical pleasure on account of the perceived logic and ‘rightness’ of its return to the tonic harmony. The realisation of the implications of the cycle affords a different, more pleasurable, kind of cognitive stimulus to the ‘irksome’ denials (albeit minimally so) theorised extensively by Meyer and Narmour (Jan 2007:127–8).

52. In particular, the octave location of pitches in the cycle is perhaps insignificant perceptually and cognitively, but has a bearing on the EMD calculation on account of the different (by twelve) MIDI numbers of a given pitch-class presented in two contexts an octave apart.

53. For tasks such as the identification of melodies related to Roslin Castle (Typke et al. 2007:166, Table 1) it would be taken as evidence of complete dissimilarity.

54. One issue not covered here is that of tempo differentials between antecedents and consequents, insofar as this is quantifiable. A consequent museme based on a duple diminution of a given antecedent which left pitch unchanged would nevertheless be perceived as identical to its antecedent if the tempo were also halved.

55. Note that I am using the term ‘conceptual similarity’ here in a somewhat different sense to that used in the discussions of Musemes c and g. Here it is taken to mean seeing beyond notational differences in order to grasp the underlying affinities between two patterns, whereas in Sections 5.2.3 and 5.2.7 such notational differences were understood as symbolic of a more significant distinction. In neither sense, however, are the conceptual differences between the forms of any of the musemes considered here so great as to undermine their status as potential evolutionary antecedents and consequents.

56. Some of this material is adapted from Jan (2007:135).

57. In such calculations the number of pitches moved is equal to the number of pitches in the museme –1, because there is nowhere to which the terminal pitch can be moved.

REFERENCES


ABSTRACT

The aim of this article is to schematise and quantify certain of the similarity relationships which are relevant to the application of memetics to music, in order to sketch a methodology by which evolutionarily significant resemblances (particularly in the melodic dimension) might be evaluated. The degree of similarity between two musical patterns is central in memetics, because the determination of whether homology (similarity resulting from replication), as opposed to analogy (similarity arising fortuitously), is operative in particular transmission situations often hinges on it. After outlining David Cope’s five categories of melodic similarity and relating them to memetics, the Earth Mover’s Distance (EMD) metric is discussed and its relevance to the psychological, evolutionary, and neurobiological aspects of similarity is evaluated. It is argued that the EMD may be used to quantify both the perceptual-cognitive salience intrinsic to musemes, and the effort required in mutating a museme from a ‘source’ (evolutionarily earlier) to a ‘copy’ (evolutionarily later) form, the latter understood as an index of similarity. These ideas are brought together by means of an analysis of a short passage from the finale of Beethoven’s Piano Sonata op. 106 Hammerklavier, which applies various weighting schemes to the EMD calculations.

Keywords: Similarity, memetics, museme, Earth Mover’s Distance, mutation.

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ABOUT THE AUTHOR

Steven Jan is Senior Lecturer in Music at the University of Huddersfield. His research interests lie in the fields of late-eighteenth and early-nineteenth century music, theory and analysis, computer-aided musicology, and the application to music of evolutionary theory. His *The Memetics of Music: A Neo-Darwinian View of Musical Structure and Culture*, the first book-length exposition of this subject, was published in 2007. He has published articles in *Music Analysis*, the *International Journal of Musicology*, the *Journal of the Royal Musical Association*, *Computer Music Journal*, *Musicae Scientiae*, *Music Theory Online*, and *Interdisciplinary Science Reviews*.

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