PARTICLE TRACKING STUDIES USING DYNAMICAL MAP CREATED FROM FINITE ELEMENT SOLUTION OF THE EMMA CELL*

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Abstract

The possibility of transverse displacement of the magnets in the EMMA non-scaling FFAG [1] and the unconventional size of the accelerator motivate a careful study of particle behavior within the EMMA ring. The magnetic field map of the doublet cell is computed using a Finite Element Method solver; particle motion through the field can then be found by numerical integration. However, by obtaining an analytical description of the magnetic field and using a differential algebra code to integrate the equations of motion, it is possible to produce a dynamical map in Taylor form. This has the advantage that, after once computing the dynamical map, multi-turn tracking is far more efficient than repeatedly performing numerical integrations. Also, the dynamical map is smaller (in terms of computer memory) than the full magnetic field map; this should allow different configurations of the lattice to be represented very easily using a set of dynamical maps, with interpolation between the coefficients in different maps.

INTRODUCTION

Particle tracking studies require an accurate model of the lattice to give reliable results. However, the EMMA magnets are so short that their magnetic field is not accurately representable with conventional (e.g. hard edged) models. This paper will be divided into three parts, explaining the main steps from magnets design to beam dynamics. First, production of tables of field values (by numerical finite element solver) is explained. Second, we describe the conversion of these tables of numerical values into analytical expressions. Finally, the motion of a particle in the cell will be simulated by means of dynamical maps.

MAGNETIC FIELD COMPUTATION

The short lengths of magnets in EMMA means that a 3D model is required to describe the magnetic field. The magnet designs [2] were developed in OPERA [3], which uses the Finite Element Method (FEM) to solve Maxwell's equations for given current and magnetic material distributions, and provides numerical field values on a grid within a specified region. We used the same software for this study, implementing mesh properties and boundary conditions more adapted to particle tracking.

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Each cell within the EMMA ring includes a horizontally focusing (F) quadrupole magnet, and a horizontally defocusing (D) quadrupole magnet. The magnetic axes of the F and D quadrupoles are not aligned, so treating the pair as a single magnetic unit, the usual four-fold symmetry of a quadrupole is broken. However, symmetry about the plane of the ring is conserved; thus, computing the upper half of the model is necessary and sufficient.

The cell is enclosed by non material borders, required for the FEM solver. Longitudinally, one can link the entrance face and the exit face by a periodic condition, since a cell is a period of the ring. EMMA consists of 42 cells; therefore, the periodicity condition involves an exit face rotation by an angle $2\pi/42$ to match the following entrance face (which is parallel to the faces of the magnets in the following cell). All the elements are aligned with respect to a straight line of length 394.481 mm, which represents one side of a 42-sided polygon. This polygon is defined so as to be close to the expected closed orbit of a particle with energy 15 MeV (the machine is designed to run electrons between 10 and 20 MeV). The vacuum chamber extends transversly a few centimetres on each side: simulations of the magnetic field and the particle motion have to be precise in this area.

The Finite Element Method is based on the principle that by solving a set of equations for field values on a finite number of points (defining a mesh), the field can be determined everywhere by interpolation. The finer the mesh, the more accurate is the field description, but the longer it takes to solve the field equations. By steadily increasing the mesh density and computing the effect on a particle trajectory, one can determine the mesh density required to give a convergent tracking behavior. Studies have shown from 5 million (5 hours solving) to 11 million mesh elements (20 hours solving), the output coordinates differ by less than 20 μm. For the present study, this precision is considered good enough and models based on field solutions with 5 million mesh elements are used.

ANALYTICAL FIELD REPRESENTATION

The values of the field on a 3D cartesian grid with 1 mm step in all directions is extracted from the FEM code as a table. This format is rather cumbersome (because of the file size) and errors within are difficult to detect. However, by performing a discrete Fourier transform (DFT), one can obtain a more convenient, analytical representation of the field, in terms of a set of Fourier coefficients. The Fourier basis functions are solutions of Maxwell's equations, and can be expressed either in cartesian coordinates, or in cylindrical polar coordinates. Given the cylindrical geometry of the system, with azimuthal symmetry $\vec{B}(\phi + 2\pi) = \vec{B}(\phi)$, cylindrical polar coordinates are more appropriate for our purposes:

$$B_\rho = \sum_{m,n} a_{mn} I'_m(nk_z\rho)\sin(m\phi)\sin(nk_z z), \qquad (1)$$
$$B_\phi = \sum_{m,n} a_{mn} \frac{m}{nk_z\rho} I_m(nk_z\rho)\cos(m\phi)\sin(nk_z z),$$
$$B_z = \sum_{m,n} a_{mn} I_m(nk_z\rho)\sin(m\phi)\cos(nk_z z).$$

The coefficients a_{mn} are obtained by performing a 2D DFT on the values of B_ρ on a cylinder of radius ρ_0, whose axis is coincident with the z axis. A given a_{mn} is simply the corresponding Fourier coefficient, normalised by $I'_m(nk_z\rho_0)$, where $I_m(r)$ is a modified Bessel function. Longitudinally, the analysis is simplified if the model edges (at $z = 0$ and $z = 394.481$ mm) are far enough from the magnets that the fringe fields vanish on both sides. As we will see, in reality, the field does not go lower than a few gauss between two cells. The Fourier representation is accurate only within the cylinder: for $\rho > \rho_0$, residual errors to the fit increase exponentially.

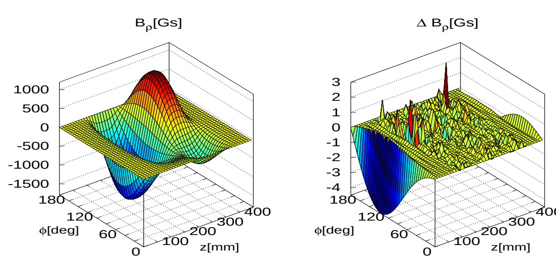

Figure 1: Radial component of the magnetic field (left) and residual of the fit (right) on a reference cylinder within one EMMA cell.

Fig. 1 shows the radial field component and the residual of the fit on a reference cylinder of radius $\rho_0 = 15$ mm in one EMMA cell. The (relatively) large residuals at the entrance and exit of the cell arise from the non-zero values of the field, which cannot be represented by the Fourier basis functions we have used. It would be possible to extend the basis functions to include these fields, however, with errors of the order of few gauss, this fit is considered good enough for our tracking studies.

Although cylindrical polar coordinates are more appropriate for the field description, beam dynamics studies are more conveniently performed using cartesian coordinates. In that case, we need to obtain a representation of the field using basis functions in cartesian coordinates. In cartesian coordinates, the vertical component (for example) of the magnetic field can be written:

$$B_y = \sum_{m,n} c_{mn} e^{i(mk_x x + nk_z z + ik_y y)}. \qquad (2)$$

Generally, a fit to numerical field data based on Eq. (2) is less successful than one using cylindrical polar coordinates, Eq. (1), because the cartesian expression has an intrinsic periodicity in the transverse variable x, which does not properly describe the field in accelerator magnets. However, it is possible to obtain a reasonable description of the field by converting the coefficients a_{mn} to a set c_{mn}. A value for k_x needs to be assumed, and can be chosen to minimise the residual in the final field description. Fig. 2 shows the residuals of the cartesian field description to the original numerical field data; the exponential increase in the residuals outside the reference cylinder originally used for obtaining the fit can be seen clearly.

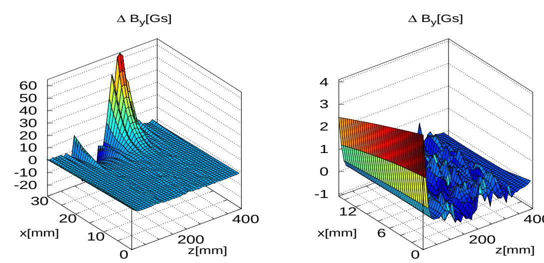

Figure 2: Residuals on the median plane after cartesian conversion over a large range in x (left), and over a region contained within the original reference cylinder (right).

GENERATION OF A DYNAMICAL MAP

To track particles in a lattice, the equations of motion have to be solved. Where a quadrupole can be represented by a hard-edged model, a first order approximation is often good enough to describe particle motion through the magnet. However, the short magnets in EMMA are dominated by the fringe field, and it is possible that nonlinear effects will be strong. A dynamical map may be obtained in the form of a power series, truncated at some order, by implementing an integrator for the equations of motion in a differential algebra code: we use COSY Infinity [4]. This requires that the magnetic field (or equivalently, the vector potential) be provided in an analytical form, for which the Fourier representation obtained as described above is appropriate. In principle, any integration routine may be used; we use the symplectic integrator developed by Wu, Forest, and Robin [5]. The phase advance per cell, dispersion, time of flight etc. are all easily obtained from the map.

To sum up, from a large table of numerical field values from OPERA, an analytical representation of the field is found, and by integrating the equations of motion through this field using a differential algebra code, one obtains a dynamical map conveniently expressed as a power series. With this dynamical map, tracking studies are easy and fast. However the region of validity of this map has to be carefully considered.

RANGE OF VALIDITY

As explained above, the magnetic field is considered to be accurate within the reference cylinder. Where the nominal trajectory of a particle is along the magnetic axis of a quadrupole, it is possible to obtain an accurate field description over (essentially) the entire physical aperture by choosing a reference cylinder that is coaxial with the quadrupole, and that just fits within the pole tips. However, in EMMA, the two quadrupoles within a cell are not coaxial; this limits the radius of the cylinder that can be used. Furthermore, since the nominal trajectory of a particle is curved within a cell, it is possible that real trajectories are not contained within the largest reference cylinder that can be constructed through a cell. This issue can be addressed in two ways. First, it is possible to perform the field fit on the surface of an elliptical cylinder corresponding approximatively to the vacuum chamber. Dragt and Mitchell have described an appropriate set of basis functions [6]; however, we have not yet implemented the required functions in COSY.

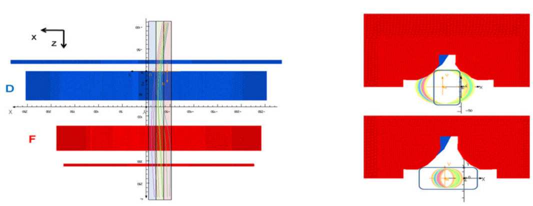

Figure 3: top right: Front view $((\vec{x},\vec{y})$ plane) of the cell with large cylinders touching the poles; bottom right: same front view with small cylinders. A larger area can be covered. Left: Top view of the cell $((\vec{x},\vec{z})$ plane) with various trajectories and three small cylinders.

Alternatively, one can use field representations derived from a set of reference cylinders, each with a different axis and radius, chosen to cover effectively the entire interior of the vacuum chamber (see fig.3). Depending on the position of the particle at each step of integration, the most appropriate cylinder is selected to determine the field. This method is studied in Fig. 4; the trajectories of a particle simulated either with a field derived from a 20 mm radius cylinder, or from two 10 mm radius cylinders, differ by 1 mm.

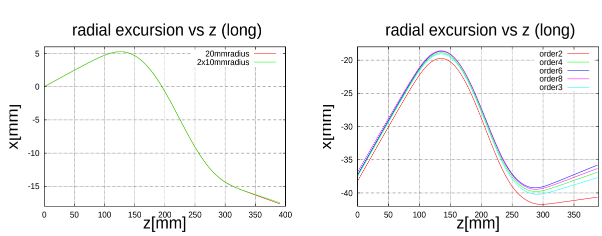

Figure 4: left:trajectory dependance on radius of the reference cylinder; right:variation of the 15MeV closed orbit with the order of the truncation of the dynamical map .

Assuming an accurate magnetic field over a sufficiently wide region, an approximation is still made when truncating the map at a particular power. For particles following trajectories far from the reference trajectory, higher order contributions may make a significant contribution, and it is difficult to compute these contributions with good accuracy. This is a particular problem when particles cover a wide range of energies in EMMA; however, since it is possible to identify a different closed orbit for each energy [7], it is also possible to define an appropriate reference trajectory in each case. Finally, it is necessary to perform convergence tests to determine the necessary order to which the map must be computed see fig. 4.)

SUMMARY AND NEXT STEPS

It is possible to construct a dynamical map corresponding to a detailed description of a magnetic field. The dynamical map provides a more efficient representation of particle behaviour in the accelerator than a numerical magnetic field map. In the case of EMMA, special features of the geometry present certain challenges in applying the general technique; however, it appears possible to address these challenges by “patching together” field maps and dynamical maps covering different regions. The next step is to determine the accuracy and reliability of the dynamical map by making comparisons with numerical integration codes, such as zgoubi [8]. The ultimate goal is to construct a model of the accelerator based on dynamical maps covering a range of machine configurations.

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