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A new method to characterize the structured tessellation surface

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Abstract

Tessellated structure surface has been widely used for engineering surface. However, comprehensive characterization method for tessellated structure surface does not exist. In this paper, a systematic method that based on lattice building combined with the spectral analysis for the characterization of tessellation surface is introduced. The basic procedure includes six steps: pre-processing the measured data; converting the filtered data to AACF domain; finding the peak and the translation vectors; building the lattice and classifying the lattice type; tessellation reconstruction and finally comparison. Experimental works verified the effectiveness of the proposed method.

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1. Introduction

High potential applications for "structured surface" related products are found in the diverse automotive, biotech, medical and consumer applications and are one of the current drivers of future economic growth. The ability to adequately characterise these structured surface geometry is crucial in the optimisation and control of such functional surfaces. Surface metrology as a discipline is thus currently undergoing a huge paradigm shift: from stochastic surfaces to structured surfaces [1,2].

In traditional surface creation processes, such as grinding, milling, polishing, honing etc., the surface features and the organisation of the surface features are stochastic in nature. Even in turned surfaces that have a fundamental periodic component there is still a large stochastic element in the surface due to the material shear mechanisms, ploughing and smearing at the toolworkpiece interface. Surfaces with a dominant stochastic feature pattern are termed "stochastic surfaces". Stochastic model have been widely used for the analysis of stochastic surfaces [1,2].

Surfaces with a dominant deterministic feature pattern are termed "structured surfaces". All structured surfaces are specifically designed to meet a highly defined functional requirement such as the specific optical response of a Fresnel lens. The dominant deterministic feature pattern can be repeated patterns, designed structures, or randomly generated functional topography. A key common feature of structured surfaces is that they are normally high aspect ratio surfaces where the surface geometry is the critical determination of component function. The scale of structured surfaces ranges from the micro scale down to the nano scale. Specific examples include MEMS/NEMS devices, micro molding, micro fluidic systems (lab on a chip), structured coatings, microlenses and optic elements, bio integration coatings, self cleaning coatings and sheet metal products.

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Tessellation surfaces are a typical type of structured surfaces: these are surfaces that have a repeated structure comprising of a collection of tiles that fills the plane with no overlaps and no gaps. Following terms refer to the same: tessellated structured surface/tessellated surface/tessellations. The most widely used tessellated engineering surfaces are optical lenses: such as the retroreflection used on road surfaces, road signs and safe clothing; micro-lens array for back light optics etc. Other widely used tessellation surface in engineering is the Electron Beam Textured steel sheet surface which have repeated pattern on its surface; and abrasive surfaces (typical example is the 3M TRIZAC abrasive paper surface) which consists of an array of triangular based micron sized pyramids.

Structured surfaces are made up of discrete objects (primitive cells). Hence, the definitions and characterisation of these nano-scale structured surfaces is at the interface between the discrete and continuous worlds, which is very difficult to evaluate using traditional stochastic models. Traditional surface texture parameters, such as roughness and waviness, cannot be related to the function for structured surface; and statistical descriptions add little information. For example: diffraction grating behaviours is basically determined by the ratio of groove spacing to optical wavelength and by the groove geometry. Such surface finish parameters as both profile parameters (Rt, Rg, etc.) and areal parameters (Sq. etc.), by contrast, provide little information in this case. For tessellation surface it is very often that the shape and geometry information is more important than surface texture. For example, when evaluate the functional quality of a micro lens array (MLA), the geometry of the individual features (individual lens), and their maximum, minimum, and mean values, and the distribution of the features (spacing of the lens) are more important than the surface roughness. Conventional linear filtration technologies based on the wavelength/frequency extract the useful spatial information not efficiently.

In this paper, a new approach that based on the spectra analysis combined with lattice building for the characterization of the tessellation surface is proposed. In the second section, the strategy and the scheme structure of the proposed method will be introduced, and then in section 3 details of the technique for each step will be described; case studies are also demonstrated in section 4; finally, section 5 will give the discussions and conclusions.

2. Methodology

There are two basic elements to define a tessellation surface: (1) Primitive structures: can also be cited as single-tessellate tile, which is the basic structures(building blocks) that form the tessellation surface; (2) the repeating rules that describe how the primitive structure will be repeated. Correspondingly, there are three basic objectives which need to be reached for the characterization of the tessellation surface: (1) classification of different tessellation surfaces; (2) identification of the basic tessellate tiles' attributes; and the definition of the relationships between tessellate tiles in a stable and robust way.

To reach above objectives, two different types of methodology can be used. One is bottom-up method and the other is top-down method. In the bottom-up method, primitive structures (single-tessellation tiles) of a tessellation are firstly to be extract. The spatial relationships of these features are then analysed and stored in a data structure according to their spatial relationship. Finally, the periodicity is obtained by extracting the two independent translation vectors. As the bottom-up approach starts with feature-extraction process, it is in general sensitive to distortions in the texture, and also the feature exaction methods (such as the motif analysis, morphology analysis, pattern analysis) are very complicated and time-consuming. In contrast, the top-down approaches extract the texture structure before feature-extraction process is performed. Fig. 1 shows the scheme structures of the two different methodologies. In this paper, the author will use the topdown method.

As for the top-down approaches, Conners et al. have used a parallelogram to describe the primitive structure of the tessellations. In their method, the two sides of the parallelogram were used to represent the two translation vectors that describe repeating patterns of the tessellation [3]. Zuker and Terzopoulos use cooccurrence matrix (CM) method to extract the primitive structure [4]. However, in their method only one translation vector can be extracted, which result in that the locations of the primitive structure cannot be located correctly. Kim and Park improved the speed of the CM based method by using the projection information [5]. However, all CM based-methods are computationally



Fig. 1. (a) bottom-up method; (b) top-down method



Fig. 2. Structure of the proposed method to characterization tessellation surface

expensive because the CM features are computed for all possible orientations and resolutions [6]. Fourier transform has been used to extract periodicity of a tessellation. In Fourier transform based methods, the peaks in the Fourier spectrum are used to characterize the periodicity of tessellation. However, it is difficult to extract correct periodicity of a tessellation because peaks in the Fourier spectrum of the practical measured tessellation surface are usually not significant and thus difficult to find. To overcome the above weakness in top-down approach, Hsin-Chih Lin et al. reported an autocorrelation function based method to extracting periodicity of a regular texture [6]. In this method, autocorrelation function is used as the basis of a texture measure, and peaks in the autocorrelation function characterize the texture periodicity. Compared with the peaks in the Fourier spectrum, the peaks in the autocorrelation function are prominent and thus easier to find. However, their method has limited use as the peaks in the ACF will be dominated by the form and form error and the peaks that reflect the pattern of the tessellation will be weakened and not easy to find.

To characterize the tessellation surface, the authors using a lattice based method combined with the spectral analysis to characterization the tessellation surface. Fig.2 gives the structure of the proposed method. The basic procedure includes six steps. In the first step, preprocessing the original measured data is needed. The aim of this step is to remove the irrelevant information from the tessellations that are to be characterized. A regression filter technique is used for this purpose as this method can remove form very well while not changing the tessellations itself. In the second step, the filtered data is converted into the transfer domain to make the relationship of the tessellated tile more significant. Our method here is the Areal Autocorrelation Function (AACF) of the surface as it has been proved a good representation for the repetition over planar space. This method can highly improve the significance of the peaks in the AACF that represent the periodicity of the tessellations thus it can improve the accuracy of the extraction. In the third step, the peak points of the AACF are linked and then by using a statistic histogram method two main linearly independent translation vectors, which can attribute the relationship of the tessellation tiles, can be constructed. In the fourth step: linking the vectors a lattice grid can be built. And from the relationship of the length and the angles between the two sides of the lattice, the lattice can be classified as: Parallelogram, rectangular, rhombic, square, and hexagonal. Accordingly, tessellated surfaces can be categorized by the above lattice types. In the fifth step, the 'mean tessellation' or the 'reference tessellation' will be built according to the extracted primitive structures and the translation vectors. And finally, in the sixth step, compare the pre-processed tessellation with the reconstructed tessellations, the 'roughness' parameters can be calculated.

3. Techniques

3.1. Pre-processing by using robust regression filter

Measured surfaces are always needed to be filtered before any further analysis, because the measured surface always have some irrelevant information that we are not interested in. For example, form and form error that not part of the tessellations. Conventional linear filter such as the linear Gaussian filter and the linear spline filter are not suitable for the tessellated structure surface analysis. This is because that the tessellated surfaces are always have high aspect ratio surfaces and the height distribution is not Gaussian distribution or not near symmetric distribution, which are one of the basic requirement when these linear filter can be used. When linear filter used for the tessellated surface, not only the distortion at the boundary and the departure of the reference surface from the measured surface, but also features (primitive structure) themselves will be distorted.

$$\arg\min_{w_{i,j}} \left\{ \sum_{l=0}^{n-1} \sum_{k=0}^{m-1} \rho \left(z_{k,l} - w_{i,j} - \sum_{h=0}^{2} \beta_{h,2-h}(i,j) ((k-i)\Delta x)^{h} ((l-j)\Delta y)^{2-h} \right) \cdot s_{i,j,k,l} \right\}$$
(1)

(2)

Consequently, in this paper the author use the nonlinear regression filter to pre-process the tessellation surface. The robust regression filter was originally introduced by Seewig [7]. The authors in this paper introduced an M-estimation based general model and the fast algorithm of the robust regression filter both for profile and areal surface analysis [8]. The general model of the robust regression filter up to 2^{nd} order is as following. Let z_{ij} , w_{ij} be the measured surface data and resulting mean surface data respectively, i, j be the index of the surface coordinate in x and y direction respectively and i = 0, ..., m-1; j = 0, ..., n-1, m, n be the number of points in x and y direction respectively, and the lateral sample spacing are $\Delta x, \Delta y$. The discrete model of the 2nd order areal Gaussian regression filter can be described the minimisation problem, see Equ (1), where

$$s_{i,j,k,l} = (1/\alpha\lambda_{cx})\exp\left(-\pi\left((k-i)\Delta x/\alpha\lambda_{cx}\right)^{2}\right)$$

$$\cdot(1/\alpha\lambda_{cy})\exp\left(-\pi\left((l-j)\Delta x/\alpha\lambda_{cx}\right)^{2}\right)$$

and $\lambda_{cx} \lambda_{cy}$ are the selected cutoff wavelength in x and y direction respectively. The re-weighting function $\rho(\cdot)$ is a symmetric, positive-definite function with a unique minimum at zero. The resulting reference surface can be obtained by solving the following linear equations iteratively:

$$\begin{split} & \boldsymbol{w}^{(\prime)}(i,j) = \\ & \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} G_{00}^{(\iota-1)} & G_{10}^{(\iota-1)} & G_{20}^{(\iota-1)} & G_{11}^{(\iota-1)} & G_{22}^{(\iota-1)} \\ G_{10}^{(\iota-1)} & G_{20}^{(\iota-1)} & G_{11}^{(\iota-1)} & G_{22}^{(\iota-1)} & G_{12}^{(\iota-1)} \\ G_{01}^{(\iota-1)} & G_{11}^{(\iota-1)} & G_{02}^{(\iota-1)} & G_{12}^{(\iota-1)} & G_{03}^{(\iota-1)} \\ G_{20}^{(\iota-1)} & G_{21}^{(\iota-1)} & G_{21}^{(\iota-1)} & G_{21}^{(\iota-1)} & G_{22}^{(\iota-1)} \\ G_{11}^{(\iota-1)} & G_{21}^{(\iota-1)} & G_{12}^{(\iota-1)} & G_{21}^{(\iota-1)} & G_{22}^{(\iota-1)} \\ G_{02}^{(\iota-1)} & G_{12}^{(\iota-1)} & G_{31}^{(\iota-1)} & G_{22}^{(\iota-1)} \\ G_{11}^{(\iota-1)} & G_{21}^{(\iota-1)} & G_{22}^{(\iota-1)} & G_{13}^{(\iota-1)} & G_{12}^{(\iota-1)} \\ G_{02}^{(\iota-1)} & G_{12}^{(\iota-1)} & G_{22}^{(\iota-1)} & G_{13}^{(\iota-1)} & G_{12}^{(\iota-1)} \\ \end{bmatrix} \begin{bmatrix} D_{11}^{(\iota-1)} \\ D_{20}^{(\iota-1)} \\ D_{20}^{(\iota-1)} \\ D_{20}^{(\iota-1)} \\ D_{20}^{(\iota-1)} \\ \end{bmatrix} \end{bmatrix}$$

where:

$$G_{u,v}^{(t-1)}(i,j) = \sum_{l=0}^{n-1} \sum_{k=0}^{m-1} \delta_{k,l}^{(t-1)} \cdot ((k-i)\Delta x)^u \cdot ((l-j)\Delta y)^v \cdot s_{i,j,k,l}$$
(3)

$$D_{u,v}^{(t-1)}(i,j) = \sum_{l=0}^{n-1} \sum_{k=0}^{m-1} \delta_{k,l}^{(t-1)} z_{k,l} \cdot ((k-i)\Delta x)^u \cdot ((l-j)\Delta y)^v \cdot s_{i,j,k,l}$$
(4)

and *t* is the iteration step, $\delta^{(t)}(r) = \rho'(r)/r$ is the reweighting function with *r* being the residual from last iteration $r_{i,j} = z_{i,j} - w_{i,j}^{(t-1)}$.

Fig. 3 and Fig. 4 show two typical results by using the robust regression filter to process the tessellation surfaces. Compared with the linear filter, the robust regression filter has the advantages: (1) it can process surfaces with significant form, and can also act as a form filter; (2) all boundary data can be processed and used with no distortion; (3) the shape of the feature can be well preserved, for example, the shapes and geometries of the individual laser zone spots in the LZT surface in Fig. 3 and the individual micro lens in the Fig. 4.

3.2. AACF

The autocorrelation function is a very useful tool for processing random signals. It describes the general dependence of the topographical values at one position on the topographical values at another position. For areal surface evaluation, it can not only describe the spatial relation dependences of the surface topography, but also describe the direction and periodicity of the surface texture. In the authors' previous research, the AACF has been used to describe different machining methods as each has very different surface textures patterns and consequently very different patterns for their AACF



Fig. 3. (a) measured LZT surface; (b) tessellation surface by using proposed regression filter



Fig. 4. (a) measured micro lens array surface; (b) tessellation surface by using proposed regression filter

spectra [9]. Furthermore, the AACF spectra can reflect the texture periodicity and directionality more clearly than visualisation of a surface topography map. In this research, the AACF is used to measure the repeating rules of the tessellation surfaces.

The AACF is defined as the average of a function multiplies with the function itself translated, describing thus the inter-dependence of the surface heights at different positions. The AACF is defined mathematically as:

$$R(\tau_x, \tau_y) = E\Big[\eta(x, y)\eta(x + \tau_x, y + \tau_y)\Big]$$

$$= \lim_{l_x, l_y \to \infty} \frac{1}{4l_x l_y} \int_{-l_y}^{l_y} \int_{-l_x}^{l_x} \eta(x, y)\eta(x + \tau_x, y + \tau_y) dxdy$$
(5)

where $\eta(x_k, y_l)$ is a filtered surface data set after the form has been removed. The discrete AACF takes the form:

$$R(\tau_{i},\tau_{j}) = \frac{1}{(M-i)(N-j)} \sum_{l=1}^{N-j} \sum_{k=1}^{M-i} \eta(x_{k},y_{l})\eta(x_{k+i},y_{l+j})$$

 $i = 0,1,...,m < M;$
 $j = 0,1,...,n < N;$
 $\tau_{i} = i \cdot \Delta x; \tau_{i} = j \cdot \Delta y$
(6)

The AACF of a surface signal has three properties: (1) Symmetry, $R(\tau_i, \tau_j) = R(\tau_{-i}, \tau_{-j})$; (2) The maximum value is at the central point; (3) Similar pattern and periodicity as the surface texture.

There is FFT based fast algorithm can be used to calculate the AACF very efficiently:

$$R(\tau_i, \tau_j) = IFFT \left(FFT \left(\eta(x_k, y_l) \right) FFT^* \left(\eta(x_k, y_l) \right) \right)$$
(7)

where, FFT represents fast Fourier transform, *IFFT* represents the inverse Fourier transform, and * represent the complex conjugate. Fig 5(a) and fig 6(a)are two measured standard calibration block surface. For these surfaces, mean feature spacing (mean spacing between blocks) is one of the important parameters need to calculated. To calculate these parameters by the conventional way, each individual blocks need to be separated firstly and then centers of these blocks need to be found. However, as mentioned previously, separating the individual blocks need time-consuming algorithm, and define centers of these block can only get inaccurate results due to the noise and the irregularity of the each block. Fig 5(b) and Fig 6(b) are the AACFs of the measured surfaces. It is very clear that the AACF peaks can significantly represent the periodicity of the tessellations. One can obtain the feature distance by simply measure the distance between the peaks in AACF. The other advantage of this method is that the result is robust against noise and the shape deviation of



Fig. 5. .(a) measured calibration block;(b)AACF



Fig. 6. (a) measured micro lens array surface; (b) tessellation surface by using proposed regression filter

individual feature due to the AACF naturally has an "average" effect.

3.3. lattice building

Four basic planar translational symmetries can be used to categorize the tessellation surface: (1) Translations, means one can slide a pattern along a certain direction with certain distance and it will fall back upon itself with all the patterns exactly matching; (2) Rotations: A rotation fixes one point in the place and turns the rest of it some angle around that point. The angle of rotation can only be 180° , 120° , 90° , 60° ; (3) Reflection: A reflection fixes one line in the plane, called the axis of reflection, and exchanges points on one side of the axis with point on the other side of the axis at the same distance from the axis; (4) Glide reflection: It is composed of a reflection across an axis and a translation along the axis. According to these four basic planar translational symmetries, a wall paper group method has been used to classify the tessellations. However, this method is very complex and trivial for engineering surfaces as some different types are actually very similar. It is very difficult to distinguish them using computer for practical engineering surface.

In this study, the authors use the simplified method, called lattice method, to characterize the tessellations of engineering surfaces. For any point, the collection of translates of it by translation symmetries of a pattern forms a lattice. There are five different kinds of lattice: (1) *Parallelogram*: if a lattice has a parallelogram as a fundamental region, it is parallelogram lattice. A

parallelogram lattice's symmetry group has translations and half-turns, but there are neither reflections nor glidereflections. (2) Rectangle: if a lattice has a rectangle as a fundamental region, it is rectangle lattice. Rectangular lattice is a special parallelogram lattice (90° parallelogram). A rectangular lattice's symmetry group has translation, half-turns and reflections. (3) Rhomb: if a lattice has a rhombic as a fundamental region, it is a rhombic lattice. (A rhombus is a parallelogram with equal sides). A rhombic lattice's symmetry group has translation, half-turns, reflections and glide reflections. (4) Square: If a lattice has a square fundamental region, it is called a square lattice. Square lattice is very special rhombic lattice (90° rhombus). A square lattice's symmetry group has translation, rotations of 90°, 180°, reflections and glide reflections. (5) Hexagon: if a lattice has a 60° rhombus as a fundamental region, it is called a hexagonal lattice. That's because in that case, the points in the lattice nearest any one point in the lattice are the vertices of a regular hexagon. Hexagonal lattice is a very special rhombic lattice (60° rhombus). A hexagonal lattice's symmetry group has translation, rotations of 60°, 120°, and 180°, reflections and glide reflections. The fundamental region can be spanned by two

displacement vectors. The generalized Hough Transform (GHT) has been used to find two displace vectors from the peaks of the AACF. This pair of displacement vectors is then used to construct the primitive structure (fundamental region) of the tessellations, and from the relationship of the length and the angles between the two displace vectors the type of the tessellations can be classified accordingly.

4. Case studies

To test effectiveness and efficiency of the present method, a number of different types of simulated and measured tessellation surface have been processed. Some typical data are selected in this paper to demonstrate the results. Fig. 7(a) is the measured EBT surface. One can intuitively see that tessellations do exist in the surfaces, but it is still difficult to quantify it (what is the primitive structure? and how is it repeated?) from the original surface. By using the presented method, one can clear see in Fig. 7(c), the peaks in the AACF can represent the periodicity of the tessellations, and direction and length of two translation vectors can show how the primitive structure is repeated. Using the above



mentioned method, this EBT surface can also be classified as Hexgon type tessellation surface. Following Fig. 8 and Fig. 9 are 3M traizac abrasive paper surface and micro needles array surface respectively. By using the presented method, they can be classified as rectangle tessellations and the primitive structure and the repeating rules can be recognized by the two displacement vectors.

5. Conclusions

Tessellated structure surfaces have been widely used for engineering surface. However, systematic characterization systems for tessellated structure surfaces do not exist. In this paper, a new top-down strategy that based on lattice building combined with the spectral analysis is introduced. The structure of the proposed method basically includes six steps: preprocessing the measured data to remove the irrelevant information; convert the filtered data to transfer domain (AACF) to make the relationship of the tessellated tile more significant; thirdly, finding peaks in AACF and obtain the translation vectors; fourthly, building the lattice and extract the primitive structure; and the last two steps are tessellation reconstruction and comparison. This paper shows the first 4 steps. Some initial experimental works have shown that the proposed method can effectively and efficiently analyze the structure of the tessellation surface. However, for comprehensive characterization tessellations the last two steps are needed. Then it is natural that our further work will be concentrated on the reconstruction of the ideal model/design model of tessellations. The initial solution for this purpose is repeating the extracted texture primitives according to two displacement vectors. And then compare the filtered surface with the reconstructed surface, the deviation can be obtained for further analysis.

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