University of Huddersfield Repository

Johnson, Anthony and Unver, Ertu

The Conceptual Design of a Kinetic Energy Storage Device to Store 20 KWh of Energy

Original Citation


This version is available at http://eprints.hud.ac.uk/13812/

The University Repository is a digital collection of the research output of the University, available on Open Access. Copyright and Moral Rights for the items on this site are retained by the individual author and/or other copyright owners. Users may access full items free of charge; copies of full text items generally can be reproduced, displayed or performed and given to third parties in any format or medium for personal research or study, educational or not-for-profit purposes without prior permission or charge, provided:

- The authors, title and full bibliographic details is credited in any copy;
- A hyperlink and/or URL is included for the original metadata page; and
- The content is not changed in any way.

For more information, including our policy and submission procedure, please contact the Repository Team at: E.mailbox@hud.ac.uk.

http://eprints.hud.ac.uk/
Report
On Behalf of
ESP

The Conceptual Design of a Kinetic Energy Storage Device To Store 20 KWh of Energy

by

School of Computing and Engineering, Mechanical Engineering

Ertu Unver  PhD, MSc, PG Cert, BSc (Hons)
School of Art, Design and Architecture, 3D Digital & Product Design

Dates:
Phase 1: 9/May/2011 Completion and presentation of stage 1: page 1 to 22.
Phase 2: on-going research page 23 to 45)

This report is strictly confidential and owned by ESP and University of Huddersfield. It may not be published, in full or in part, without the consent of the Authors and ESP Energy Storage ltd.
Contents (Phase 1)

Contact Details 3

Introduction 4

Conceptual Design Exploration 5

Target Specification and Requirements 6

Design Considerations and Decisions 7

Rim Type 7

Disc Type Flywheel 10

Primary Bearing System 12

Secondary Bearing System 13

Containment 13

Motor Generator Set 14

Control Equipment 14

Machine Monitoring 15

Magnetic Drive Coupling 15

Electrical Aspects Design and Analysis 15

Conclusions and Recommendations 16

References 18

Appendix-1 19-22
<table>
<thead>
<tr>
<th>Contents (Phase 2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 2 of the Flywheel project</td>
<td>23</td>
</tr>
<tr>
<td>Additional team members joining to the research team</td>
<td>23</td>
</tr>
<tr>
<td>Design Evolution from Concept 1</td>
<td>24</td>
</tr>
<tr>
<td>Design Concept 2, Evolving to Concepts 3 and 4</td>
<td>25</td>
</tr>
<tr>
<td>Concept 5</td>
<td>30</td>
</tr>
<tr>
<td>Concept 6</td>
<td>32</td>
</tr>
<tr>
<td>Conclusion and further plan</td>
<td>36</td>
</tr>
<tr>
<td>Appendix 2</td>
<td>37-43</td>
</tr>
</tbody>
</table>
Contact details:

ESP Storage Products Limited details:

Contact Person: Terry Idle, CEO, ESP Energy Storage Products Limited

Tel: 07506 553663
email: terry.idle@esp.im

ESP is a company registered in England and Wales (7392042). Registered Offices: Horley Green House Horley Green Road, Claremont, Halifax, England, HX3 6AS

University staff contacts:

Dr Ertu Unver, Senior Lecturer
Tel 01484483848
Email: e.unver@hud.ac.uk
University of Huddersfield, School of Art, Design and Architecture
Huddersfield, HD1 3DH
MA 3D Digital Design course leader
3D Subject Area - BA/BSc Product, Transport
Web sites: www.huddersfield3d.co.uk, www.hud.ac.uk

Anthony Johnson, BSc (Hons) M.I.Mech.E. C.Eng.
Email: a.d.johnson@hud.ac.uk
Web: www.1719design.com
Mobile: 07800 947196
University of Huddersfield, School of Computing and Engineering
Queensgate, Huddersfield, HD1 3DH
Introduction

This conceptual design was commissioned by ESP with the express intention of creating a viable concept for a Kinetic Energy Storage Device (KESD) in the form of a flywheel system. The intention is to glean energy from the national grid when electricity is very low cost and return it to the national grid when energy is more expensive. As an example; one kWh of energy bought in the early morning when demand is low costs approximately £20. If this could be stored until a high demand period, say early evening, the cost would be approximately £250 per kWh. If several KESD’s could be run there would be potential of making a great deal of money. On a national basis; several thousand KESD’s could reduce the necessity for so many power stations. KESD’s could also store solar power and wind power to be used when demand for power is high.

Summary

This report presents two conceptual styles of Kinetic Energy Storage Devices as outlined as follows:

- Cylindrical, rim type flywheel system
- Solid disc type flywheel system

The general specifications are listed as follows:

<table>
<thead>
<tr>
<th></th>
<th>Rim Type Rotor</th>
<th>Disc Type Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Style</td>
<td>Hollow cylinder</td>
<td>Solid cylinder</td>
</tr>
<tr>
<td>Outer Diameter (mm)</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Inner diameter (mm)</td>
<td>450</td>
<td>n/a</td>
</tr>
<tr>
<td>Rotor depth (mm)</td>
<td>1000</td>
<td>500</td>
</tr>
<tr>
<td>Material</td>
<td>steel</td>
<td>steel</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>7500</td>
<td>7500</td>
</tr>
<tr>
<td>Rotor mass (kg)</td>
<td>930</td>
<td>1060</td>
</tr>
<tr>
<td>Angular velocity (Rev/min)</td>
<td>18000</td>
<td>18000</td>
</tr>
<tr>
<td>Surface speed (m/s)</td>
<td>565</td>
<td>565</td>
</tr>
<tr>
<td>Energy (Joules)</td>
<td>74,200,000</td>
<td>84,800,000</td>
</tr>
<tr>
<td>Energy (KWh)</td>
<td>20.61</td>
<td>23.55</td>
</tr>
<tr>
<td>Power (KW)</td>
<td>5.73</td>
<td>6.54</td>
</tr>
<tr>
<td>1 Tonne mass lift (m)</td>
<td>8150</td>
<td>8,152</td>
</tr>
<tr>
<td>Containment (Primary)</td>
<td>Steel casing</td>
<td>Steel casing</td>
</tr>
<tr>
<td>Containment (Secondary)</td>
<td>Concrete lined Pit</td>
<td>Concrete lined Pit</td>
</tr>
<tr>
<td>Bearing system radial</td>
<td>Magnetic Levitation</td>
<td>Magnetic Levitation</td>
</tr>
<tr>
<td>Bearing System (secondary)</td>
<td>Rolling element Brgs</td>
<td>Rolling element Brgs</td>
</tr>
<tr>
<td>Motor Generator Drive coupling</td>
<td>Magnetic</td>
<td>Magnetic</td>
</tr>
<tr>
<td>Chamber Type</td>
<td>Vacuum</td>
<td>Vacuum</td>
</tr>
</tbody>
</table>
Conceptual Design Exploration

A great deal of research was applied to the project in order to gain an overview of the technology involved with flywheel design. The most useful of the references are listed in the reference section pp17.

Several basic styles of flywheel solutions were examined before the final concepts were selected. At the initial briefing solutions were aired as possible concepts as follows:

- Toroidal flywheel system supported on bearings at the rim
- Containment system using a secondary rotor to absorb the energy from a bursting flywheel.

**Toroidal Flywheel:**

A schematic can be seen in figure 1. This rotor system is supported on magnetic levitation bearings at the rim. The motor / generator system is also sited at the rim. Though the system is sound in principle, investigations revealed certain practical difficulties. The design would incorporate a solid (or multiple disc) rotor where the windings would be built into the stationary casing. The high surface speeds at the periphery would use eddy currents which are known to be only 70% efficient. Furthermore magnetic levitation would be difficult to control and expensive to apply at the larger peripheral diameter.

![Figure 1: Schematic of a Toroidal Rotor](image)

This solution was discarded due to its elevated expense and lead time in developing the system.

**Containment System Using a Secondary Rotor**

This is quite a feasible solution to the problem of absorbing the energy of a bursting rotor. It represents one of the three generally accepted containment systems and is often used for containment for flywheels used in vehicles, aeroplanes, etc. It was considered for use in the current concept and would certainly work. Its main drawback however is its increased complexity and subsequent extra cost. The current concept is ground based and therefore can use a static, passive containment system where weight (mass) is not a problem; the solution was therefore discarded in favour of a less expensive containment system.
The initial approach was to design the system using standard technology with the minimal new technological development. It was hoped that this would keep costs down and make manufacturing relatively simple. The target specification is outlined below:

**Target Specification and Requirements**

**Research**

The research revealed that high speed burst was a large problem. If the flywheel system disintegrated there would be large pieces of material possessing massive amounts of energy, being thrown around. Essentially the device would have enough energy to be equivalent to a large bomb if not contained properly. Many variations of flywheel have been designed. Some of the major research in this area has been in flywheel construction. When a flywheel bursts the main danger is from the high velocity chunks of material. Much research is being applied to the design of rotors in fibre reinforced resin. When these burst they disintegrate into fibrous elements which do not possess any great mass.

**Target Specification**

The initial target specification was defined through initial research combined with the original specification from ESP

- **Envelope size:** 1m³
- **Power rating:** 20 to 50KWh
- **Efficiency:** > 75%
- **Power degradation over 24hrs:** < 10%
- **Calendar life:** 10 years
- **Max sound power level:** 63dBA
- **Low speed:** approx 20k rev/min

**Design Approach**

The analysis, shown on the spreadsheets in the appendix, was iterated several times for each style of flywheel until optimum sizes were found. The analysis revealed some enormous parameters which needed to be carefully considered in the design.

Furthermore the parameters focussed the design in particular directions. An example of this is the surface speed parameter of 565m/s. This is almost twice the speed of sound and demands some means of reducing the turbulence and noise within the enclosure.

The progress of the design became an iterative approach where parameters such as diameter, mass, speed, etc were varied until a reasonable output emerged. It was difficult to decide on the most appropriate type of flywheel, rim type or disc type so a concept of both versions has been proposed.
Design Considerations and Decisions

Flywheel Design

Rim Type

The rim type flywheel takes the form of a large rotating cylinder. The main advantage is that many of the elements which allow the spin, e.g. bearings, can be built inside the cylinder creating a more compact overall unit. Stresses in the cylinder can be considered to be hoop stresses only rather than radial and hoop stresses as with a solid flywheel. Careful consideration needs to be applied to the link between shaft and flywheel since this would be subjected to some high stresses. Finite element stress analysis would be used to verify strength.

The parameters selected for the rim type flywheel are as follows in table 1. The rim type flywheel concept can be seen in figure 2.

<table>
<thead>
<tr>
<th>Rim Type Rotor</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Style</td>
<td>Hollow cylinder</td>
</tr>
<tr>
<td>Outer Diameter (mm)</td>
<td>600</td>
</tr>
<tr>
<td>Inner diameter (mm)</td>
<td>450</td>
</tr>
<tr>
<td>Rotor depth (mm)</td>
<td>1000</td>
</tr>
<tr>
<td>Material</td>
<td>steel</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>7500</td>
</tr>
<tr>
<td>Rotor mass (kg)</td>
<td>930</td>
</tr>
<tr>
<td>Angular velocity (Rev/min)</td>
<td>18000</td>
</tr>
<tr>
<td>Surface speed (m/s)</td>
<td>565</td>
</tr>
<tr>
<td>Energy (Joules)</td>
<td>74,200,000</td>
</tr>
<tr>
<td>Energy (KWh)</td>
<td>20.61</td>
</tr>
<tr>
<td>Power (KW)</td>
<td>5.73</td>
</tr>
<tr>
<td>1 Tonne mass lift (m)</td>
<td>8150</td>
</tr>
<tr>
<td>Containment (Primary)</td>
<td>Steel casing</td>
</tr>
<tr>
<td>Containment (Secondary)</td>
<td>Concrete lined Pit</td>
</tr>
<tr>
<td>Bearing system radial</td>
<td>Magnetic Levitation</td>
</tr>
<tr>
<td>Bearing System (secondary)</td>
<td>Rolling element Brgs</td>
</tr>
<tr>
<td>Motor Generator Drive</td>
<td>Magnetic</td>
</tr>
<tr>
<td>coupling</td>
<td></td>
</tr>
<tr>
<td>Chamber Type</td>
<td>Vacuum</td>
</tr>
</tbody>
</table>

Table 1: Rim-type Flywheel Concept Specification
Figure 2: Proposed Rim Type Flywheel
Disc Type Flywheel

The disc type flywheel takes the form of a solid steel cylinder of dimensions Diameter 600mm x 500mm long. Though this is physically smaller than the rim type flywheel the overall package including the casing and motor/generator set is a little longer. This is because the bearings have to be set above and below the rotor rather than inside as with the rim type rotor.

Radial and hoop stresses are important considerations. Hoop stresses are a maximum at the outer rim but radial stresses are a maximum near the centre of the cylinder. Consideration therefore is needed when fitting the rotor to the shaft since extra stresses near the centre should be avoided. The concept uses a particular method of fixing to the shaft which is hoped will prove to be successful. Careful stress analysis needs to be done to ensure that this method, or any other selected method will be satisfactory. The parameters selected for the disc type flywheel are as follows in table 2. The disc type flywheel concept can be seen in figure 3.

<table>
<thead>
<tr>
<th>Disc Type Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Style</strong></td>
</tr>
<tr>
<td><strong>Outer Diameter (mm)</strong></td>
</tr>
<tr>
<td><strong>Inner diameter (mm)</strong></td>
</tr>
<tr>
<td><strong>Rotor depth (mm)</strong></td>
</tr>
<tr>
<td><strong>Material</strong></td>
</tr>
<tr>
<td><strong>Density (kg/m³)</strong></td>
</tr>
<tr>
<td><strong>Rotor mass (kg)</strong></td>
</tr>
<tr>
<td><strong>Angular velocity (Rev/min)</strong></td>
</tr>
<tr>
<td><strong>Surface speed (m/s)</strong></td>
</tr>
<tr>
<td><strong>Energy (Joules)</strong></td>
</tr>
<tr>
<td><strong>Energy (KWh)</strong></td>
</tr>
<tr>
<td><strong>Power (KW)</strong></td>
</tr>
<tr>
<td><strong>1 Tonne mass lift (m)</strong></td>
</tr>
<tr>
<td><strong>Containment (Primary)</strong></td>
</tr>
<tr>
<td><strong>Containment (Secondary)</strong></td>
</tr>
<tr>
<td><strong>Bearing system radial</strong></td>
</tr>
<tr>
<td><strong>Bearing System (secondary)</strong></td>
</tr>
<tr>
<td><strong>Motor Generator Drive coupling</strong></td>
</tr>
<tr>
<td><strong>Chamber Type</strong></td>
</tr>
</tbody>
</table>

Table 2: Disc Type Rotor Specification
Figure 3: Proposed Disc Type Flywheel
Primary Bearing System

The bearing system is required to hold the rotor in place whilst allowing rotation around 20,000 rev/min with the minimal frictional resistance. Several options were considered:

- Magnetic levitation bearings
- Rolling element bearings
- Gas film bearings
- Fluid film bearings (journal)

Gas film bearings require a pressurised gas to be constantly pumped between the shaft and bearing. In a vacuum environment this would be inappropriate and the option was discarded.

Fluid film bearings are essentially oil filled. Upon rotation of the journal (shaft) the oil is pulled into the contact/support area and acts as a cushion. Frictional resistance is low but a constant flow of fluid is required requiring shaft seals within the vacuum chamber. It was thought that the frictional resistance applied by the seals would create too much drag and reduce efficiency. There may also be a problem with heat generation within the fluid at high speeds.

Magnetic levitation bearings are frictionless and can operate within a vacuum environment. Their major disadvantage is that they require a control system and that the magnetic coils require power. Research shows however that many KESD’s use magnetic levitation as a primary bearing system. It was discovered that SKF could produce the magnetic levitation bearings required and also have experience in producing motor generator equipment which could operate at these high speeds.

SKF provided some very useful data which was used directly in the concept designs. They gave approximate sizes of bearing units and suggested how the layout should be applied. Beacon power is a company in the United states for which SKF have previously designed such bearings. Please see their web site [www.beaconpower.com](http://www.beaconpower.com).

A typical radial Magnetic Levitation Bearing can be seen in figure 4.

![Figure 4: Typical Radial Magnetic Levitation Bearing](image-url)
The axial bearings should be sited at the top of the housing. The magnetic flux applied is attractive allowing the rotor to hang from the bearing. This arrangement can be seen in both figures 2 and figure 3.

**Secondary Bearing System**

It is evident from the research and from advice given by SKF that there needs to be a secondary bearing back-up system. This is necessary should power fail to the magnetic levitation bearings. SKF recommended high speed ball bearings for this particular role. These bearings would function as radial and axial bearings. Figure 5 shows typical ball bearings. The application can be seen in the concept designs in figures 2 and 3.

![Figure 5: Typical Ball Bearings](image)

**Containment**

The containment system has to be applied to two aspects of the design: containment of the vacuum and containment of a bursting flywheel.

There are several approaches to containment of a bursting flywheel.

- Brute force approach using heavy-walled pressure vessels
- Spinning ring which soaks up the energy of the impacting flywheel debris
- Energy absorbing liner. The soft catch approach

The containment system requires design effort to be applied. When the flywheel is at running speed a great deal of energy is contained. This energy is enough to vertically lift a 1 tonne mass 8.1 km. Put another way there is energy contained in the flywheel equivalent to 18kg (40lbs) of TNT.

Generally it is thought that a combined approach might be the way forward. This would entail a light casing to contain the vacuum and a heavier casing such as a concrete, perhaps below ground vessel to contain any flywheel burst. This chamber might also incorporate some form of soft catch energy absorbing system which could take the form of sand or pea gravel filled chambers which break, thus allowing the contents to infiltrate the chamber when a burst occurs.

The current concept uses segmental sand bags set inside the concrete basin. If the bursting flywheel breaches the vacuum casing the sand bags will disintegrate allowing the sand to absorb the debris. The concrete casing will act as final resort containment. Please see figure 6 for a 3D impression of the containment.

Many current flywheels merely incorporate the brute force approach where a heavy casing is combined with a vacuum chamber. In the case of a burst a great deal of
energy needs to be contained so any such casing needs to be very firm and should be
mounted on heavy foundations.

Figure 6: Impression of the Segmental Sandbag and Concrete Containment System

Motor Generator Set

The motor / Generator set drives the flywheel up to speed during energy input and
converts rotational energy into electrical energy during energy extraction. Due to the
high speeds this is quite a special arrangement and requires specialist design
attention. SKF are able to provide the design expertise and design a suitable M/G set
to suit the specific design needs of the project. Typical components can be seen in
figure 7.

Figure 7: Typical Motor/ Generator Components

Control Equipment

The magnetic levitation bearings could become unstable unless constantly monitored
and controlled. The service incorporates appropriate matching and supply of
monitoring and control equipment. Figures 8 and 9 show typical control and monitoring
devices.
Machine Monitoring

Within the package SKF also offer machine monitoring software which takes input from several sensors to relay the information to a monitoring station. This information could include: rotational speed, out of balance detection, temperature, energy input efficiency, energy extraction efficiency, etc.

Magnetic Drive Coupling

The modular design of the KESD separates the motor/generator from the vacuum chamber of the flywheel. Normal engineering practice would incorporate lip seals to contain the vacuum, but the seals would be in contact with the shaft causing frictional resistance and heat generation. Ceramic seals would be required. The better solution would be to use a magnetic drive coupling.

This is a non-contacting coupling which transfers magnetic flux through a non-magnetic membrane thus allowing drive to be applied between the two halves of the coupling. One half would be situated inside the vacuum chamber; the other half inside the motor / generator chamber. A typical magnetic drive coupling can be seen in figure 11.
Electrical Aspects Design and Analysis

SKF are the current specialist source of magnetic levitation bearings. There are other companies who can offer this service, however SKF have been the most informative and helpful. They not only offer a full manufacture and design service but they also offer all the ancillary equipment needed to operate and maintain the system. Elements for the system are listed as follows:

- Design and manufacture service for magnetic levitation bearings
- Radial magnetic levitation bearings
- Axial magnetic levitation bearings
- Control system for magnetic levitation bearings
- Machine monitoring system
- Design and manufacture service for motor / generator set
- Control system for the motor / generator set

Incorporated in the total package is a high performance finite element predictive analysis tool for magnetic levitation applications.

Conclusions and Recommendations

The proposed designs show the format of both a rim type flywheel system and also a solid (disc type) flywheel system. The sizes of the rotors are generally correct though other aspects of the design are approximate. It should be noted however that during the process of conversion from concept to detailed, manufacturable units many of the details may change. There are many technical design details which require careful consideration such as methods of balancing the rotors and stress analysis of the rotors and frame. Below is a basic list of work to be accomplished before a flywheel system can be manufactured:

Further Work Required to Progress the Project

- Finite element stress analysis vacuum chamber
- Mag/Lev radial bearing design
- Mag/Lev axial bearing design
- Control system for magnetic levitation bearings
- Machine monitoring system
- Design and manufacture of motor / generator set
- Control system for the motor / generator set
- Stress analysis of rotors for burst limitation
- Fluid flow analysis within the chamber
- Vacuum pump and equipment selection
- Rolling element bearing design and selection
- Vacuum Casing design
- Explosion containment system design
- Foundations design
- Selection of materials

There are two concepts put forward, each with its own advantages and disadvantages. There needs to be a selection of either version so that detailed work can commence.
Both concept designs fit most of the initial criteria. During the detail design process some of these may need to change to accommodate more precise constraints for use or perhaps to accommodate physical parameters thrown up by the development process. The concept specifications are as follows in table 3.

**Industrial Concept**

The KESD system has been designed for an industrial application where banks of KESD’s can be housed. The appendix shows several 3D pictures which indicate scale and the probable industrial setting. The soft-catch and concrete containment system is also shown.

<table>
<thead>
<tr>
<th></th>
<th>Rim Type Rotor</th>
<th>Disc Type Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Style</strong></td>
<td>Hollow cylinder</td>
<td>Solid cylinder</td>
</tr>
<tr>
<td><strong>Outer Diameter (mm)</strong></td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td><strong>Inner Diameter (mm)</strong></td>
<td>450</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>Rotor depth (mm)</strong></td>
<td>1000</td>
<td>500</td>
</tr>
<tr>
<td><strong>Material</strong></td>
<td>steel</td>
<td>steel</td>
</tr>
<tr>
<td><strong>Density (kg/m^3)</strong></td>
<td>7500</td>
<td>7500</td>
</tr>
<tr>
<td><strong>Rotor Mass (kg)</strong></td>
<td>930</td>
<td>1060</td>
</tr>
<tr>
<td><strong>Angular velocity (Rev/min)</strong></td>
<td>18000</td>
<td>18000</td>
</tr>
<tr>
<td><strong>Surface Speed (m/s)</strong></td>
<td>565</td>
<td>565</td>
</tr>
<tr>
<td><strong>Energy (Joules)</strong></td>
<td>74,200,000</td>
<td>84,800,000</td>
</tr>
<tr>
<td><strong>Energy (kWh)</strong></td>
<td>20.61</td>
<td>23.55</td>
</tr>
<tr>
<td><strong>Power (kW)</strong></td>
<td>5.73</td>
<td>6.54</td>
</tr>
<tr>
<td><strong>1 Tonne mass lift (m)</strong></td>
<td>8150</td>
<td>8,152</td>
</tr>
<tr>
<td><strong>Containment (Primary)</strong></td>
<td>Steel casing</td>
<td>Steel casing</td>
</tr>
<tr>
<td><strong>Containment (Secondary)</strong></td>
<td>Concrete lined Pit</td>
<td>Concrete lined Pit</td>
</tr>
<tr>
<td><strong>Bearing System Radial</strong></td>
<td>Magnetic Levitation</td>
<td>Magnetic Levitation</td>
</tr>
<tr>
<td><strong>Bearing System (secondary)</strong></td>
<td>Rolling element Bearings</td>
<td>Rolling element Bearings</td>
</tr>
<tr>
<td><strong>Motor Generator Drive Coupling</strong></td>
<td>Magnetic</td>
<td>Magnetic</td>
</tr>
<tr>
<td><strong>Chamber Type</strong></td>
<td>Vacuum</td>
<td>Vacuum</td>
</tr>
</tbody>
</table>

Table 3: Concept Specifications for the ESP Kinetic Energy Storage Device
**Rotor Selection**

There is little to choose between the two types of rotor. The list of differing parameters is as follows:

<table>
<thead>
<tr>
<th>Rim Type</th>
<th>Disc Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Lower mass 930kg</td>
<td>- Higher mass 1060kg</td>
</tr>
<tr>
<td>- Height overall 1877mm</td>
<td>- Overall height 2028mm</td>
</tr>
<tr>
<td>- Energy 20.61 kWh</td>
<td>- Energy 23.55 kWh</td>
</tr>
<tr>
<td>- Power 5.73 kW</td>
<td>- Power 6.54 kW</td>
</tr>
<tr>
<td>- Stresses in rim only (Less prone to burst)</td>
<td>- Radial and Hoop stresses present</td>
</tr>
</tbody>
</table>

- Design and manufacturing will present a similar level of difficulty
- Balancing relatively easier with the rim-type flywheel due to increased length
- Cost implications will be similar for each type
References

Many references were consulted. These were a mixture of both technical papers and company data. The following is a list of the most useful references.

- SKF Website: www.skf.com
- Beacon Power: www.beaconpower.com
- Flywheel Energy Storage System: California Energy Commission 2004
- Compact Flywheel Energy Storage System: Koyo Seiko Company
- Flywheel Energy Storage: Riello-UPS: www.riello-Ups.co.uk
- Flywheel energy and Power Storage Systems: Boland, Bernhoff and Leijon 2004
- Designing Safer Flywheels: Steven Ashley: Test devices inc.: www.testdevices.com
- Murakami, m., (2007), japan, Title: design of an energy storage flywheel system using permanent magnet bearing and superconducting magnetic bearings
- Seong-yeol yoo, (2009) korea, Title: design of magnetically levitated rotors in a large flywheel,
- Werfel (2008), Germany (german e-on), Title: 250kW Flywheel with HTS Magnetic Bearing for Industrial use
- Bjorn Bolund (2005), sweden, Title: Flywheel energy and power storage systems
- Read T. Doucette, (2011), OXFORD Engineering Science department, UK, 2010/11, Title: A comparison of high speed flywheels, batteries and ultracapasitors and the bases of cost and fuel
- T. H. Sung, (2011), Korea, Title: Flywheel energy storage system with a horizontal axle mounted on high Tc superconductor bearings
### Flywheel Characteristics

**Disc Type Rotor**

<table>
<thead>
<tr>
<th>Dia (m)</th>
<th>Radius (m)</th>
<th>Depth (m)</th>
<th>Mass (kg/m³)</th>
<th>Volume (m³)</th>
<th>Mass (kg)</th>
<th>Ang Vel (rad/sec)</th>
<th>Surface Speed (m/s)</th>
<th>Moment of Inertia (Joules)</th>
<th>KE (KWh)</th>
<th>Power (KW)</th>
<th>1 Tonne Lift</th>
<th>Rim Stress (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.10</td>
<td>0.80</td>
<td>5700.00</td>
<td>0.03</td>
<td>188.52</td>
<td>30000.00</td>
<td>314.02</td>
<td>1.22</td>
<td>7515.84</td>
<td>40567.95</td>
<td>0.36</td>
<td>9129.21</td>
</tr>
<tr>
<td>0.30</td>
<td>0.15</td>
<td>0.70</td>
<td>5700.00</td>
<td>0.05</td>
<td>371.15</td>
<td>20000.00</td>
<td>2094.67</td>
<td>4.18</td>
<td>9100.13</td>
<td>40567.95</td>
<td>0.54</td>
<td>9129.21</td>
</tr>
<tr>
<td>0.40</td>
<td>0.20</td>
<td>0.70</td>
<td>5700.00</td>
<td>0.09</td>
<td>659.82</td>
<td>20000.00</td>
<td>2094.67</td>
<td>13.20</td>
<td>28950.40</td>
<td>72108.00</td>
<td>0.64</td>
<td>9129.21</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>0.70</td>
<td>5700.00</td>
<td>0.14</td>
<td>1030.97</td>
<td>20000.00</td>
<td>2094.67</td>
<td>32.22</td>
<td>70679.10</td>
<td>11268.76</td>
<td>0.73</td>
<td>9129.21</td>
</tr>
<tr>
<td>0.60</td>
<td>0.30</td>
<td>0.70</td>
<td>5700.00</td>
<td>0.20</td>
<td>1484.60</td>
<td>20000.00</td>
<td>2094.67</td>
<td>66.81</td>
<td>146561.65</td>
<td>16227.18</td>
<td>0.83</td>
<td>9129.21</td>
</tr>
</tbody>
</table>

**Note:** Speed of Sound = 343.2 m/s

### Rim Type Rotor

<table>
<thead>
<tr>
<th>Outer Dia (m)</th>
<th>Inner Dia (m)</th>
<th>Outer Rad (m)</th>
<th>Inner Rad (m)</th>
<th>Depth (m)</th>
<th>Mass (kg/m³)</th>
<th>Volume (m³)</th>
<th>Mass (kg)</th>
<th>Ang Vel (rev/min)</th>
<th>Surface Speed (m/s)</th>
<th>Moment of Inertia (kgm²)</th>
<th>KE (Joules)</th>
<th>Power (KW)</th>
<th>1 Tonne Mass Lift (t)</th>
<th>Rim Stress (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>0.12</td>
<td>0.06</td>
<td>0.80</td>
<td>7500.00</td>
<td>1103.38</td>
<td>15000.00</td>
<td>1571.00</td>
<td>392.75</td>
<td>3470.00</td>
<td>42819.48</td>
<td>11.89</td>
<td>3401.00</td>
<td>633874.73</td>
</tr>
<tr>
<td>0.60</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.50</td>
<td>7500.00</td>
<td>1994.15</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>44796.61</td>
<td>749661.11</td>
<td>22.08</td>
<td>613.00</td>
<td>815133.00</td>
</tr>
<tr>
<td>0.60</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.40</td>
<td>7500.00</td>
<td>795.32</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>39747.28</td>
<td>6359728.00</td>
<td>17.67</td>
<td>4913.00</td>
<td>815133.00</td>
</tr>
<tr>
<td>0.60</td>
<td>0.30</td>
<td>0.15</td>
<td>0.08</td>
<td>0.30</td>
<td>7500.00</td>
<td>506.49</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>26847.96</td>
<td>467967.96</td>
<td>18.25</td>
<td>3683.00</td>
<td>815133.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.25</td>
<td>0.15</td>
<td>0.08</td>
<td>0.70</td>
<td>7500.00</td>
<td>938.18</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>29322.00</td>
<td>520988.00</td>
<td>14.47</td>
<td>4023.00</td>
<td>566064.00</td>
</tr>
<tr>
<td>0.4</td>
<td>0.20</td>
<td>0.15</td>
<td>0.08</td>
<td>0.80</td>
<td>7500.00</td>
<td>648.04</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>377.04</td>
<td>367828.00</td>
<td>10.42</td>
<td>2913.00</td>
<td>566064.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.25</td>
<td>0.15</td>
<td>0.08</td>
<td>0.60</td>
<td>7500.00</td>
<td>804.16</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>25130.00</td>
<td>446555.00</td>
<td>12.40</td>
<td>3454.00</td>
<td>566064.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.25</td>
<td>0.15</td>
<td>0.08</td>
<td>0.65</td>
<td>7500.00</td>
<td>871.17</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>27220.00</td>
<td>483767.96</td>
<td>13.44</td>
<td>3734.00</td>
<td>566064.00</td>
</tr>
<tr>
<td>0.55</td>
<td>0.28</td>
<td>0.22</td>
<td>0.11</td>
<td>0.70</td>
<td>7500.00</td>
<td>1047.88</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>38962.00</td>
<td>704093.60</td>
<td>19.56</td>
<td>5433.00</td>
<td>684938.00</td>
</tr>
<tr>
<td>0.6</td>
<td>0.30</td>
<td>0.22</td>
<td>0.11</td>
<td>0.50</td>
<td>7500.00</td>
<td>917.86</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>4130.00</td>
<td>733958.00</td>
<td>20.39</td>
<td>5663.00</td>
<td>815133.00</td>
</tr>
<tr>
<td>0.6</td>
<td>0.30</td>
<td>0.45</td>
<td>0.23</td>
<td>1.00</td>
<td>7500.00</td>
<td>927.87</td>
<td>18000.00</td>
<td>1855.00</td>
<td>565.56</td>
<td>4175.00</td>
<td>7419638.00</td>
<td>20.61</td>
<td>5753.00</td>
<td>815133.00</td>
</tr>
</tbody>
</table>
b) Concept Renderings:

Figure 12: Single containment basin showing lid and control cabinet

Figure 13: Impression of the industrial Application (Lids removed)

Figure 14: Impression showing the Soft-Catch Segmental Sand Bags
Figure 16: Renderings of the Proposed KESD (Hollow Flywheel Version)

Figure 17: Renderings of the Proposed KESD (Solid Flywheel Version)
Phase 2: Stage 2 of the Flywheel project (9/May/2011 till now)

The crucial elements to a successful design were considered to be the bearings, radial and axial and the rotor stresses. Once these elements were finalised then the rest of the KESD could be designed to suit. Use of Bearings was investigated whilst finite element analysis was performed on the proposed rotor designs this is where other specialist team members included in the research team.

Additional team members joining to the research team

**Dr Simon Barrans (SMB)**
Specialism: Stress Analyst
Duties: Analyse the stresses in the rotor and define materials, rotor speeds and rotor shape according to safe stressing

**Mark Dales: (MD)**
Specialism: High current Electrical Engineer
Duties; Specify the motor Generator equipment and design/procure all other electrical elements

**Prof. Rakesh Mishra (RM)**
Specialism: Thermo-fluid behaviour and CFD techniques
Duties: Analyse the air flow around the rotor and determine the fluid drag

Design Evolution from Concept 1

Concept 1 explained in phase 1 was devised after extensive research of current models and practices. It was discovered that the original application of capturing cheap energy and selling it back to the grid at a higher cost is very feasible. Furthermore peak demand smoothing is possible and indeed is the very reason for the building of several large flywheel installations throughout the world. The kinetic storage of electrical energy possesses other benefits to emerging industries such as the electric car industry. Here recharging stations will be required and may possibly have kinetic energy storage.

The conclusion is therefore that research revealed a very viable market which is demanding some form of energy storage. Kinetic Energy Storage devices could easily fill that demand. Concept stage 1 was effectively a feasibility study of current technologies and the possibility of using these technologies to create a kinetic energy storage device (KESD). A feasibility report was presented to the board of ESP Ltd who then agreed to the project progressing to the phase 2.

The main thrust of the design was that it should follow standard engineering techniques in order to:
- Reduce time to market
- Reduce the development time
- Reduce development costs
- Ensure low cost manufacture
- Ensure sustainability issues were minimised
Bearings
Up to the point of the presentation in May 2011 of the feasibility report the team had been liaising with SKF who had been very helpful in providing guidance for the magnetic-levitation bearings used in Concept 1. The bearings needed development though SKF had previously provided similar bearings to the Beacon Project which is a similar facility based in California.

As soon as SKF understood the need for development work they lost interest saying they were overstretched and could not pursue the project. Another company, Mecos of Switzerland, was approached who could provide the necessary magnetic levitation units. They were very enthusiastic but required £100,000 to be deposited before development could begin. This clearly excluded them from the development. Two other companies were approached with no success.

Stress Analysis
Whilst the bearing search was being conducted SMB had been pursuing the stress analysis on the rotor and reported that a high carbon steel rotor with a speed of 18000 rev/min would burst long before it reached the operation speed. The data below in table 3 shows the rotor size possibilities related to material stress.

<table>
<thead>
<tr>
<th>Outer Radius Ro (m)</th>
<th>Radial Stress $\sigma_{\theta\theta}$ (MN/m$^2$) at centre</th>
<th>Moment of inertia/height</th>
<th>Height (m) required to give required energy at rated speed</th>
<th>Is design possible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>268.6726</td>
<td>3.016E+02</td>
<td>0.913</td>
<td>Y</td>
</tr>
<tr>
<td>0.45</td>
<td>340.0387</td>
<td>4.831E+02</td>
<td>0.570</td>
<td>Y</td>
</tr>
<tr>
<td>0.5</td>
<td>419.8009</td>
<td>7.363E+02</td>
<td>0.374</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 3: Rotor Size Possibilities for Solid Rotors at 7000 rev/min

The table above indicates the radius of the flywheel and the stress that flywheel would see in service. The moment of inertia and the height of the rotor are important since they are essential properties in the storage of kinetic energy. In general a larger Inertia value will store a larger value of kinetic energy but an increase in speed will increase the kinetic energy storage capacity in terms of speed squared.

Basic Equation for Kinetic Energy is as follows:

$$KE = \frac{1}{2} I \omega^2$$

Where:

$I$ = Moment of Inertia (kgm$^2$)

$\omega$ = Angular velocity (rad/sec)

It can be seen from the first line of table 3 that a rotor of radius 0.4m (800mm diameter) with a height of 0.913m (913mm) will have a moment of inertia of
3.016 x 10⁻² kgm². Importantly the radial stress in the rotor is 268.67 MN/m² which is approximately half of the envisaged ultimate tensile strength of the rotor material, medium carbon steel. This will give a safety factor of slightly under 2.0.

The conclusion from this analysis is that the original target of 18000 rev/min, for a solid rotor, cannot be achieved using steel or one of its alloys. The analysis shows that the speed needs to be substantially reduced to a maximum of 7000 rev/min.

**Design Concept 2, Evolving to Concepts 3 and 4**

The revelation that the rotor stresses had forced the angular velocity down to 7000 rev/min from 18000 rev/min helped to continue the theme of using standard engineering components and techniques since high speed radial rolling element bearings could now be considered and separate thrust bearings. The negative side to reducing the speed was that the rotor mass had to be increased to approximately 3 tonnes in order to increase the moment of inertia.

The reduction in speed also meant that expensive active magnetic-levitation bearings could be discarded, though it seemed reasonable to support the 3 tonnes mass of the vertical axially mounted rotor on permanent magnet thrust bearings. Figure 15 shows the concept.
Radial Bearings
Several bearing companies were contacted with a specification for radial bearings of:

7000 Rev/min
100mm bore
Radial loads of 15KN

Radial loads were evaluated by calculating the residual out-of-balance as directed by ISO 1940:1 Balancing Quality Requirements for Rigid Rotors. Bearing loads were then calculated for a particular bearing diameter.

Bearing companies contacted were as follows:

Barden UK (Part of the Schaeffler group)
SKF Luton
NSK UK

Each company suggested that the duty could be accommodated using a hybrid bearing of ceramic ball bearings and a high alloy steel race. In order to increase long life several lubrication strategies were suggested.

- Grease packed
- Oil lubrication (automatic injection)
- A five year maintenance plan
- Live wear monitoring

Uniformly Stressed Rotor Results
During the search for appropriate bearings SMB was conducting stress analysis on the rotor. The start point for the analysis was the solid rotor in Concept 2 with a speed of 7000 rev/min. The necessity to increase moment of inertia and reduce the radial stresses which were so high near the rotor axis, drove the rotor shape to that of a rim-type flywheel. Concept 3 shows the KESD with the new shape rotor. See figure 16.
Figure 16: Concept 3: KESD Showing Rim Type Flywheel as a Uniformly Stressed Rotor
Since the axis of the rotor was vertical it seemed reasonable to support the 3 tonnes axial load on a separate thrust bearing. The obvious choice was to use rolling element thrust bearings but it was considered that other low friction axial bearing alternatives should be investigated.

Air bearings were considered but since the unit was destined to run in a vacuum the continual leakage of air from the bearings in to the vacuum chamber would destroy the vacuum. Air bearings were therefore ruled out of the design.
The company that ESP Ltd had proposed as manufacturers JPS Ltd based in the Wirral, suggested permanent magnet thrust bearings as axial thrust bearings. They had previously conducted some ad-hoc experiments but had no real data to offer.

Manufacturers of permanent magnets were contacted to ascertain strengths, costs, etc. Research was also conducted as to best methods of application and installation of permanent magnets. High strength magnets are manufactured from neodymium.

Since the rotor was to be supported by permanent magnets the flywheel rim edges were proposed as bearing points. Consideration was given to the shape of the magnets. Cylindrical magnets were an obvious choice but research showed that heat would be generated as rotor mounted magnets passed over base mounted magnets. It soon became clear that specially shaped magnets would be required to reduce the number of “edges” within the magnetic flux and hence reduce the generated heat.

Manufacturing shaped magnets offered several options. The thrust bearing could be applied as in Concept 3, figure 16 or perhaps as in Concept 4, see figure 17. This particular configuration was devised after a quote from Best Solution Consultancy Ltd who quoted a repulsing magnet outer diameter of 540mm and an inner diameter of 160mm with a magnet thickness of 40mm. The material would be neodymium rare earth.

This high strength, high cost, sintered magnet offered several benefits:

- Reasonable gap between thrust plates of approximately 5mm
- Able to support 3 tonne mass
- Zero friction (no Contact)
- Zero maintenance

There were, however several negatives:

- Very High cost (approx £28,000 per bearing)
- Requirement to magnetise at the assembly site
- Requirement for mechanical handling equipment to enable unpacking and assembly. (due to inherent attractive forces)
- High cost of ancillary equipment

Due to the high cost of the magnetic bearing material and the anticipated high cost of the handling equipment and other ancillary equipment, it was decided that though an exceedingly viable technical option, the costs were prohibitive. The use of permanent magnet thrust bearings was discarded in favour of more traditional bearings such as rolling element thrust bearings.
Concept 5

The specification for the rotor was as follows:

- Rotor mass 3000kg
- Rotor speed 7000 rev/min
- Vertical axis

Investigations into rolling element thrust bearings quickly revealed that even hybrid ceramic bearings would be hard pressed to support 3000kg running at 7000 rev/min. A quote from one of the bearing manufacturers, SKF UK Ltd, suggested that such a bearing would match their ACD/HC super precision angular contact hybrid bearing (ceramic balls). This would need to be lubricated with an oil-air (spray) system.

Unfortunately this bearing system would only have a predicted life of 284 hours. This was the best bearing SKF could offer. Other companies offered similar bearings. They suggested that the heavy load and the rotation speed were the main problems.

It was evident that the rotor required redesigning to fulfil the design brief of storing 20KWh but with a much lower mass and a lower speed. Since the uniformly stressed rotor was essentially a rim type rotor analysis was conducted for a rim type flywheel with a spoke and shaft system to be added later within the stress analysis exercise. Revisiting the basic Kinetic Energy equation it was discovered that there were several variables which could be manipulated as follows:

\[
KE = \frac{1}{2} I \omega^2
\]

Where:
- \( I \) = Moment of Inertia (kgm\(^2\))
- \( \omega \) = Angular velocity (rad/sec)

But \( I = mk^2 \)

Where
- \( m \) = mass (kg) and \( k \) = radius of gyration (m)

giving

\[
KE = \frac{1}{2} m k^2 \omega^2
\]

Or

\[
KE = \frac{1}{2} m \left( R_2 + R_1 \right)^2 \omega^2
\]

Where:
- \( R_1 \) = outer radius (m)
- \( R_2 \) = inner radius (m)
From analysis of basic flywheel characteristics, see appendix, the kinetic energy to deliver 20KWh is 72.5MJ. For analysis purposes this value was fixed as well as the aim of a notional 1500kg for the rotor mass.

The four variables were therefore

\[ R_1 = \text{outer radius (m)} \]
\[ R_2 = \text{inner radius (m)} \]
Rotor depth (m)
\[ \omega = \text{angular velocity (rad/sec)} \]

A spreadsheet tool was used to vary the parameters which are shown in table 4 below

<table>
<thead>
<tr>
<th>Outer Dia (m)</th>
<th>Outer Rad (m)</th>
<th>Inner Dia (m)</th>
<th>Inner Rad (m)</th>
<th>Depth (m)</th>
<th>Density (kg/m^3)</th>
<th>Volume (m^3)</th>
<th>mass (kg)</th>
<th>Ang Vel (Rev/min)</th>
<th>Ang Vel (rad/sec)</th>
<th>Surface Speed (m/s)</th>
<th>Moment of Inertia kgm^2</th>
<th>KE (Joules)</th>
<th>KE (KWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.90</td>
<td>1.4</td>
<td>0.70</td>
<td>0.2</td>
<td>7500</td>
<td>0.20</td>
<td>1508</td>
<td>4000</td>
<td>419</td>
<td>377</td>
<td>611</td>
<td>53,599,690</td>
<td>14.89</td>
</tr>
<tr>
<td>1.8</td>
<td>0.90</td>
<td>1.5</td>
<td>0.75</td>
<td>0.25</td>
<td>7500</td>
<td>0.19</td>
<td>1458</td>
<td>4000</td>
<td>419</td>
<td>377</td>
<td>591</td>
<td>51,820,013</td>
<td>14.39</td>
</tr>
<tr>
<td>1.8</td>
<td>0.90</td>
<td>1.6</td>
<td>0.80</td>
<td>0.4</td>
<td>7500</td>
<td>0.21</td>
<td>1602</td>
<td>4000</td>
<td>419</td>
<td>377</td>
<td>649</td>
<td>56,949,671</td>
<td>15.82</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.6</td>
<td>0.80</td>
<td>0.2</td>
<td>7500</td>
<td>0.23</td>
<td>1697</td>
<td>3000</td>
<td>314</td>
<td>314</td>
<td>848</td>
<td>41,874,758</td>
<td>11.63</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.6</td>
<td>0.80</td>
<td>0.25</td>
<td>7500</td>
<td>0.28</td>
<td>2121</td>
<td>3000</td>
<td>314</td>
<td>314</td>
<td>1060</td>
<td>52,343,448</td>
<td>14.54</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.7</td>
<td>0.85</td>
<td>0.25</td>
<td>7500</td>
<td>0.22</td>
<td>1635</td>
<td>4000</td>
<td>419</td>
<td>419</td>
<td>817</td>
<td>71,729,910</td>
<td>19.92</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.75</td>
<td>0.88</td>
<td>0.3</td>
<td>7500</td>
<td>0.22</td>
<td>1657</td>
<td>4000</td>
<td>419</td>
<td>419</td>
<td>828</td>
<td>72,699,233</td>
<td>20.19</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.8</td>
<td>0.90</td>
<td>0.36</td>
<td>7500</td>
<td>0.21</td>
<td>1612</td>
<td>4000</td>
<td>419</td>
<td>419</td>
<td>806</td>
<td>70,721,814</td>
<td>19.64</td>
</tr>
<tr>
<td>2.2</td>
<td>1.10</td>
<td>1.8</td>
<td>0.90</td>
<td>0.2</td>
<td>7500</td>
<td>0.25</td>
<td>1885</td>
<td>3000</td>
<td>314</td>
<td>346</td>
<td>1141</td>
<td>56,298,286</td>
<td>15.64</td>
</tr>
<tr>
<td>2.2</td>
<td>1.10</td>
<td>1.9</td>
<td>0.95</td>
<td>0.25</td>
<td>7500</td>
<td>0.24</td>
<td>1812</td>
<td>3000</td>
<td>314</td>
<td>346</td>
<td>1096</td>
<td>54,099,134</td>
<td>15.03</td>
</tr>
<tr>
<td>2.2</td>
<td>1.10</td>
<td>2</td>
<td>1.00</td>
<td>0.3</td>
<td>7500</td>
<td>0.20</td>
<td>1485</td>
<td>4000</td>
<td>419</td>
<td>461</td>
<td>898</td>
<td>78,817,600</td>
<td>21.89</td>
</tr>
</tbody>
</table>

Table 4: Spreadsheet Analysis tool to Vary Flywheel Parameters

During the iteration process the outer diameter was fixed and the depth and the inner radius was varied. The angular velocity was also varied to achieve the kinetic energy value of approximately 73MJ

The rotor of:
2.2m OD, 2.0m ID, 0.3m depth, emerged as an excellent choice having a mass of only 1485kg and a KE of 78.8MJ.

The rotor of:
2.0m OD, 1.7m ID, 0.25m depth was a little heavier at 1635kg but was smaller in outer diameter. This was desirable since a smaller outer diameter would mean an overall smaller unit. In comparison there is not much to choose between either of the rotors.

The rotor of 1.8m OD, 1.6m ID, 0.4m depth, emerged as the best of the 1.8 diameter rotors but was lacking in the value of kinetic energy storage at only KE of 57.9MJ. This size of rotor was clearly inferior to the 2.0m and 2.2m diameter rotors.
Shown in figure 18 is the rim type rotor which was used to base the analysis whose results are displayed in table 4. Please note that for analysis purposes the spokes and shaft have been omitted, being relatively insignificant compared to the rim of the rotor.

Figure 18: Rim of Flywheel as Calculated for 2.0m Device

Concept 5 is shown in Figure 19. The rotor has a diameter of 2.0 m and is hollow to reduce weight and hence bearing load. It was intended that the hollow halves of the rotor would be machined separately and joined using industrial adhesive. The key turned in to the joint at the rim would key the two halves and allow them to expand radially as one component when rotating at speed.

Figure 19: Concept 5 Rim Type Flywheel

Concept 6

Intensive stress analysis revealed that the use of normally available engineering materials provided severe restrictions on the shape, speed and size of the flywheel. Furthermore the variables hitherto used were complicated to juggle to obtain the
correct rotor format. Research revealed that energy density was a comparative parameter that would give the optimum energy capacity for various rotor shapes.

**Energy Density**
Further research [16] led the design in the direction of using “Energy Density” as a useful factor in identifying the most appropriate flywheel shape as explained below:

Energy density is essentially the value of energy in Joules per kg mass of rotor.

\[
\text{Energy Density} = \frac{\text{Joules}}{\text{Kg}}
\]

\[
\begin{align*}
\text{KE} &= \frac{1 \times I \times \omega^2}{2} \quad \text{...............1} \\
\text{KE} &= \frac{m \times r^2 \times \omega^2}{2} \quad \text{...............2}
\end{align*}
\]

where:
- \(\text{KE} = \) Kinetic Energy (Joules)
- \(I = \) polar 2\(^{nd}\) moment of Inertia (kgm\(^2\))
- \(I = m \times r^2\)
- \(m = \) mass (kg)
- \(r = \) mean radius (m)
- \(\omega = \) angular velocity (rad/sec)

From equation 2 it can be seen that a reduction in angular velocity \(\omega\) must be compensated by an increase in radius \(r\).

Another parameter which is useful to consider is that of “energy density” and is shown below:

From equation 2

\[
\begin{align*}
\text{KE} &= \frac{m \times r^2 \times \omega^2}{2} \quad \text{...............2}
\end{align*}
\]

Divide both sides by mass \(m\).

\[
\begin{align*}
\frac{\text{KE}}{m} &= \frac{m \times r^2 \times \omega^2}{2m} \quad \text{...............3}
\end{align*}
\]

giving

\[
\begin{align*}
\frac{\text{KE}}{m} &= \frac{r^2 \times \omega^2}{2} \quad \text{...............4}
\end{align*}
\]

or Energy Density.

This parameter allows comparison between flywheel shapes. The new task then was to arrange the flywheel shape to give a large value of energy (Joules) for each kg of rotor mass.
The use of Energy Density quickly showed that the best shape for a rotor system was not a rim type flywheel or a cylindrical flywheel but a tapered rotor section as shown in figure 20.

![Figure 20: Concept 6 Tapered disc](image)

Stress analysis revealed that the major stresses were radial and imposed high stresses near the shaft. Essentially centrifugal forces tend to pull the flywheel material away from the shaft.

![Figure 21: Finite Element Stress Analyses Showing a Small Disc Segment by Dr Simon Barrans](image)

The diagram shown in figure 21 shows a small disc segment rotated at 500 rad/sec (4776 rev/min). The disc shows stresses 237 MN/m² in the blue area at the outside of the disc and 463 MN/m² at the centre. Should high strength materials be used these stresses are not particularly excessive and show clearly where the highest stresses are likely to occur, which is towards the centre where the disc joins the shaft.

The next step was to attach the disc to the shaft in order to determine stresses at the joint of the disc and the shaft. The analytical diagram is shown in figure 22.

![Figure 22: Analytical Segment Showing High Stress at Disc/shaft Junction by Dr Simon Barrans](image)
The concept 6 design emerged as a constant stress device which took the form of a tapered cross section fatter nearer the shaft and tapering to the outside edge. The surface, however, has been curved to enable constant stress across the section. The whole concept can be seen in figure 23.

Concept 6 parameters are as follows:

- **Outer radius**: 1.0 m
- **Outer Diameter**: 2.0 m
- **Thickness on central axis**: 110 mm
- **Thickness on outer rim**: 35 mm
- **Mass**: 1.2 to 1.5 tonnes
- **Speed**: 3650 to 4000 rpm
- **Energy storage**: 40 to 54 MJ

![Figure 23: Concept 6: Uniformly Stressed Rotor Design](image)

**Bearing Selection**

Various bearing manufacturers were contacted requesting quotations and a specification of bearing. Since the bearings required were off standard and therefore required specialist selection procedures the bearing companies were relied upon to specify bearings which could contend with the following parameters:

- **Working speed**: 4000 rev/min
- **Test speed**: 7000 rev/min
- **Rotor mass**: 2000 kg
- **Life**: 10 years

HB Bearings LTD of Honley, West Yorkshire are specialist bearing manufacturers. They declined to quote a price since the bearings could be purchased off the shelf from some of the large bearing manufacturing companies as is outlined below. They did, however assist greatly in specifying bearing types as follows:

**Radial Bearings**
- Hybrid type angular contact bearing at 15° - 25° angle of contact,
- Ø 100mm x Ø 150mm x 24mm
- To P4 run out and dimensional tolerances,
- Obtainable as sealed or not sealed
- Commercially available bearing from SKF, FAG, NSK/RHP

**Thrust Bearing**

This posed a challenge with the high running speed required since a bearing of a similar size to the radial bearings would not have the thrust capability to operate at the required speed. To be able to run at the required speed a smaller bearing needs to be considered. This would reduce the linear speed of the balls within the bearing.

The specification suggested follows:
- Hybrid type ball bearing at
- Ø 50mm x Ø 70mm x 14mm single row ball thrust bearing
- Load ratings of Ca 27 KN and Coa 75KN
- Working speeds of 4300 rev/min maximum
- Test speed 6300 rev/min limiting speed

Further research and engineering analysis have been currently carried out by Engineering staff especially vacuum, fluid dynamics, air resistance, stability, friction and vibration and noise.

**Conclusion and further plan:**

In this report an overview of advanced flywheel feasibility study is presented in the phase one. This report covered the literature review, competitive products on the market and the concept development of two alternative products with basic calculations. After presenting this report in May 2011 to ESP directors and managers the team visited manufacturing facility of ESP near Liverpool, and ESP decided to fund the development of two prototypes and raise further funding which is handled by the Research & Enterprise Centre of the University.

The team then continued to further engineering design calculation, manufacturing details, optimisation, cost of off the shelf external parts, design and detailing of the new parts to be built. And further discussion taken place with the manufacturers of the standard parts. This process was very useful as it revealed that due to the required final cost of the KESD set by ESP ltd, further design changes was necessary as the companies initially approached could not deliver the required spec parts, therefore further calculation and design iterations required.

At this point team also extended to include Dr Simon Barrans who calculated stress on the rotor and material specification. Then Mark Dales from Electrical Engineering started on electric motor generator specification, which is still on going. Also Prof. Rakesh Mishra agreed and will be calculating and advising on the rotor air frictions who specialise on Thermo-fluid behaviour and CFD techniques when the final decision is made.

The next phase of the project is to start purchasing off the shelf parts an also manufacturing and testing which we require to use manufacturing facilities of ESP.

The research team appreciate Dr Simon Barrans (SMB) on the his expert advice on the analysis of the stresses in the rotor materials, shape and specifications so far.
## Appendix 2:

### Flywheel Characteristics Iteration Spreadsheets

#### Disc Type Rotor

<table>
<thead>
<tr>
<th>Dia</th>
<th>Radius</th>
<th>Depth</th>
<th>Density</th>
<th>Volume</th>
<th>mass</th>
<th>Ang Vel</th>
<th>Ang Vel</th>
<th>Surface</th>
<th>Moment</th>
<th>KE</th>
<th>KE</th>
<th>Power</th>
<th>1 tonne</th>
<th>Rim</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>m</td>
<td>kg/m³</td>
<td>m³</td>
<td>kg</td>
<td>rpm</td>
<td>rad/sec</td>
<td>M/s</td>
<td>kg/m²</td>
<td>Joules</td>
<td>KWh</td>
<td>KW</td>
<td>Mass lift</td>
<td>N/m²</td>
</tr>
<tr>
<td>0.20</td>
<td>0.10</td>
<td>0.80</td>
<td>7500.00</td>
<td>0.03</td>
<td>188.52</td>
<td>30000.00</td>
<td>3142.00</td>
<td>314.20</td>
<td>0.94</td>
<td>4,652,751</td>
<td>1.29</td>
<td>0.36</td>
<td>2515.84</td>
<td>4,056,795</td>
</tr>
<tr>
<td>0.30</td>
<td>0.15</td>
<td>0.70</td>
<td>7500.00</td>
<td>0.03</td>
<td>371.15</td>
<td>20000.00</td>
<td>2094.67</td>
<td>314.20</td>
<td>4.18</td>
<td>9,160,103</td>
<td>2.54</td>
<td>0.71</td>
<td>2515.84</td>
<td>4,056,795</td>
</tr>
<tr>
<td>0.40</td>
<td>0.20</td>
<td>0.70</td>
<td>7500.00</td>
<td>0.09</td>
<td>659.82</td>
<td>20000.00</td>
<td>2094.67</td>
<td>418.93</td>
<td>13.20</td>
<td>28,950,450</td>
<td>8.04</td>
<td>2.23</td>
<td>4472.61</td>
<td>7,212,080</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25</td>
<td>0.70</td>
<td>7500.00</td>
<td>0.14</td>
<td>1030.97</td>
<td>20000.00</td>
<td>2094.67</td>
<td>523.67</td>
<td>32.22</td>
<td>70,679,810</td>
<td>19.63</td>
<td>5.45</td>
<td>6988.45</td>
<td>11,268,876</td>
</tr>
<tr>
<td>0.60</td>
<td>0.30</td>
<td>0.70</td>
<td>7500.00</td>
<td>0.20</td>
<td>1484.60</td>
<td>20000.00</td>
<td>2094.67</td>
<td>628.40</td>
<td>66.81</td>
<td>146,561,653</td>
<td>40.71</td>
<td>11.31</td>
<td>10063.37</td>
<td>16,227,181</td>
</tr>
</tbody>
</table>

#### Rim Type Rotor

<table>
<thead>
<tr>
<th>Dia</th>
<th>Outer Dia</th>
<th>Inner Dia</th>
<th>Diameter</th>
<th>Density</th>
<th>Volume</th>
<th>mass</th>
<th>Ang Vel</th>
<th>Moment</th>
<th>KE</th>
<th>KE</th>
<th>Power</th>
<th>1 tonne</th>
<th>Rim</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>kg/m³</td>
<td>m³</td>
<td>kg</td>
<td>rpm</td>
<td>kg/m²</td>
<td>Joules</td>
<td>KWh</td>
<td>KW</td>
<td>Mass lift</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

Note: Speed of Sound = 343.2m/s

<table>
<thead>
<tr>
<th>Dia</th>
<th>Outer Dia</th>
<th>Inner Dia</th>
<th>Diameter</th>
<th>Density</th>
<th>Volume</th>
<th>mass</th>
<th>Ang Vel</th>
<th>Moment</th>
<th>KE</th>
<th>KE</th>
<th>Power</th>
<th>1 tonne</th>
<th>Rim</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>kg/m³</td>
<td>m³</td>
<td>kg</td>
<td>rpm</td>
<td>kg/m²</td>
<td>Joules</td>
<td>KWh</td>
<td>KW</td>
<td>Mass lift</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

Note: Speed of Sound = 343.2m/s

## Flywheel Characteristics

### Disc Type Rotor

- Note: Speed of Sound = 343.2m/s

<table>
<thead>
<tr>
<th>Dia</th>
<th>Radius</th>
<th>Depth</th>
<th>Density</th>
<th>Volume</th>
<th>mass</th>
<th>Ang Vel</th>
<th>Ang Vel</th>
<th>Surface</th>
<th>Moment</th>
<th>KE</th>
<th>KE</th>
<th>Power</th>
<th>1 tonne</th>
<th>Rim</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>m</td>
<td>kg/m³</td>
<td>m³</td>
<td>kg</td>
<td>rpm</td>
<td>rad/sec</td>
<td>M/s</td>
<td>kg/m²</td>
<td>Joules</td>
<td>KWh</td>
<td>KW</td>
<td>Mass lift</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

### Rim Type Rotor

- Note: Speed of Sound = 343.2m/s

<table>
<thead>
<tr>
<th>Dia</th>
<th>Outer Dia</th>
<th>Inner Dia</th>
<th>Diameter</th>
<th>Density</th>
<th>Volume</th>
<th>mass</th>
<th>Ang Vel</th>
<th>Moment</th>
<th>KE</th>
<th>KE</th>
<th>Power</th>
<th>1 tonne</th>
<th>Rim</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>kg/m³</td>
<td>m³</td>
<td>kg</td>
<td>rpm</td>
<td>kg/m²</td>
<td>Joules</td>
<td>KWh</td>
<td>KW</td>
<td>Mass lift</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

Note: Speed of Sound = 343.2m/s

---

### Note

- Speed of Sound = 343.2m/s
- Mass lift = 13,144,017 kg
- KE = 4,652,751 Joules
From ISO1940:1

\[ U_{per} = \frac{9549 \times G \times W}{N} \text{ (gmm)} \]

Where: \( G \) = Balance Grade \( \text{G6.3} \)  \( W \) = Rotor Mass \( 3000\text{kg} \)  \( N \) = Service Speed \( 7000\text{rev/min} \)

\[ U_{per} = \frac{9549 \times 6.3 \times 3000}{7000} = 25782 \text{ gmm} \]

Find Centrifugal Force

First Find Eccentric Mass

\[ U_{per} = 25782 \text{ grm} \]

Radius of Bearing = 55mm

\[ U_{per} = e_{mass} \times \text{radius} \]

\[ e_{mass} = \frac{U_{per}}{\text{radius}} = \frac{25782}{55} = 470\text{grm or 0.47kg} \]

Find Centrifugal Force

\[ F_c = m \times \omega^2 \times r \]

Where: \( m \) = rotor mass of 3000kg  
\( \omega \) = angular velocity = \( \frac{7000 \times 2 \times \pi}{60} = 733 \text{ rad/sec} \)

\( R \) = bearing radius = 55mm or 0.055m

\[ F_c = 3000 \times 733^2 \times 0.055 = 13890 \text{ N or equivalent mass = 1416kg} \]

And is the radial force applied to the bearing.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Rotor Speed</th>
<th>Angular Velocity</th>
<th>Rotor Mass</th>
<th>Bearing Radius</th>
<th>Residual Unbalance</th>
<th>Residual Eccentric Centrifugal Force at Brg</th>
<th>Centrifugal Force at Brg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rev/Min</td>
<td>rad/sec</td>
<td>Kg</td>
<td>mm</td>
<td>g-mm</td>
<td>kg</td>
<td>N</td>
</tr>
<tr>
<td>6.3</td>
<td>7000</td>
<td>733</td>
<td>3000</td>
<td>55</td>
<td>25782.3</td>
<td>0.47</td>
<td>13857.6</td>
</tr>
<tr>
<td>2.5</td>
<td>7000</td>
<td>733</td>
<td>3000</td>
<td>55</td>
<td>10231.1</td>
<td>0.19</td>
<td>5499.0</td>
</tr>
<tr>
<td>6.3</td>
<td>4000</td>
<td>419</td>
<td>1635</td>
<td>55</td>
<td>24589.9</td>
<td>0.45</td>
<td>4315.6</td>
</tr>
</tbody>
</table>
Determination of Energy Storage Efficiency

- Determination of Drag due to Skin Friction
- Determination of Drag due to Bearing Friction
- Determination of Retardation and Efficiency

Determination of Drag due to Skin Friction

The general drag force equation shown below, was used.

\[ F_{\text{drag}} = \frac{1}{2} \rho \cdot v^2 \cdot A \cdot C_D \]

Where:
\( \rho \) = gas density (kg/m\(^3\))
\( V \) = velocity (m/s)
\( A \) = Cross sectional Area (m\(^2\))
\( C_D \) = Drag coefficient (dimensionless)

The drag coefficient was determined from the following equation after determining the Reynolds Number for the conditions. These calculations can be seen in the appendix page xxx.

\[ C_D = 1.33(\text{Re})^{1/2} \text{ per side} \]

Ref: [19]

Where: \( \text{Re} \) = Reynolds Number

\[ \text{Re} = \frac{\rho V L}{\mu} \]

Where:
\( \rho \) = fluid density (kg/m\(^3\))
\( V \) = average velocity (m/s)
\( L \) = Travelled length of fluid (hydraulic Diameter) (m)
\( \mu \) = dynamic viscosity (Ns/m\(^2\))

Parameters: from speed of sound analysis
Pressure = 200mbar
\( \rho_{\text{air}} = 1.2 \times 0.2 = 0.24\text{kg/m}^3 \)

Find Dynamic Viscosity \( \mu \)

Viscosity at 1000 mbar = 18.27 \( \mu \text{Pa s} \)
Viscosity at 200mbar = 18.27 \times 10^{-3} \times 0.2 = \mu = 3.654 \times 10^{-3} \text{Pa s}

Find \( L \) (travelled fluid length)

Normally for a pipe but the equivalent diameter can be calculated from the Hydraulic Diameter as follows:

\[ D_H = 4A \]
Where
A = cross sectional Area (m$^2$)
P = whetted surface distance (m)

Average thickness = 90mm at 448mm = average radius of disc

Cross Sectional Area = average thickness x average radius

CSA = 0.073 x 0.94 = 0.0686 m$^2$

Find Whetted Length P
Length of surface area of section
Shaft = 0.1 x 2 = 0.2
Surface = 0.94 x 2 = 1.88
Circumference = 0.036

2.116 m

Find Hydraulic Diameter $D_H$

$D_H = \frac{4A}{P} = \frac{4 \times 0.0686}{2.116} = 0.13m$

Find Average Velocity
Average occurs at average radius $r$ = 1.0

$\frac{1.0}{(2)^{1/2}} = 1.0 \times 0.707 = 0.707m$

Find Average Velocity V
From tabulated data
Velocity at circumference = 420m/s at 4000rev/min

Velocity Mean = $V = 420 \times 0.707 = 300m/s$

Find Reynolds Number

$Re = \frac{\rho VL}{\mu} = \frac{0.24 \times 300 \times 0.13}{3.654 \times 10^{-3}} = 2.561$
Find Skin Friction Coefficient $C_D$

$$C_D = 1.33(Re)^{-1/2} \text{ per side}$$

Ref: [19]

\[ C_D = 1.33(2.561)^{-1/2} \times 2 = 0.0525 \]

Find Drag Force

$$F_{\text{drag}} = \frac{1 \times \rho \times v^2 \times A \times C_D}{2}$$

$$F_{\text{drag}} = 1 \times 0.24 \times 300^2 \times 0.0686 \times 0.0525 = 38.89 \text{N Average force at average radius}$$

Find Average Torque Due to Skin Friction

$$T = \text{force} \times \text{radius}$$

$$T = 38.89 \times 0.707 = 27.5 \text{Nm}$$

Determination of Drag due to Bearing Friction

The rolling element bearings are hybrid bearings with ceramic balls. Their equivalent coefficient of friction is rated at 10% of that of a standard rolling element bearing. Since standard rolling element bearings typically have a coefficient of friction of 0.01, the coefficient of friction of ceramic hybrid bearings can be considered to be 0.001.

General Data

- Mass of rotor: 1500 kg
- Rotational speed: 4000 rev/min

Radial Bearings

- Effective diameter: 110 mm
- Radial force (N): 1571 N (gleaned from balancing forces for a rigid rotor)
- Coefficient of friction: 0.001 ($\mu$)

Thrust Bearings

- Effective diameter: 60 mm
- Axial force (N): 1571 N
- Coefficient of friction: 0.001 ($\mu$)

Find Friction Force $F_R$

Radial Bearings

NB! 2 Bearings

$$F_R = \mu N$$

$$F_R = 2 \times 0.001 \times 1571 = 3.142 \text{N}$$
Find Resisting Torque (Radial)

\[ T = \text{force} \times \text{radius} \]

\[ T = 3.142 \times 0.055 = 0.173 \text{Nm} \]

Thrust Bearings

NB! 2 bearings sharing the load

Load = 14.7KN
Each bearing will sustain 7.35KN Load.

\[ F_R = \mu N \]

\[ F_R = 2 \times 0.001 \times 7.35 \times 10^3 = 14.7 \text{N} \]

Find Resisting Torque (Axial)

\[ T = \text{force} \times \text{radius} \]

\[ T = 14.7 \times 0.03 = 0.441 \text{Nm} \]

Total Torque

\[ T_T = \text{Skin Friction} + \text{radial bearing} + \text{axial Bearing} \]

\[ T_T = 27.5 + 0.173 + 0.441 = 28.114 \text{Nm} \]

Determine Deceleration

From Newton’s second law

\[ I = \text{825kgm}^2 \text{ from data tool} \]

\[ T = I \times \alpha \]

\[ \alpha = \frac{T}{I} = \frac{28.114}{825} = 0.034 \text{m/s}^2 \text{ retardation} \]

The value of the resistance from fluid friction is significantly higher than the resistance from the bearings. Bearings contribute little to the inefficiencies whereas the skin friction is comparatively large.

Determine the Angular Velocity after 1 hour of Deceleration \( \omega_1 \)

Initial Angular Velocity \( \omega_0 = 419 \text{ rad/sec} \)
Time: 1 hour \[ t = 3600 \text{ seconds} \]
Deceleration \[ \alpha = 0.034 \text{m/s}^2 \]

From

\[ \omega_1 - \omega_0 + \alpha t \]

\[ \omega_0 = \omega_1 - (\alpha \times t) = 419 - (0.034 \times 3600) = 297 \text{ rad/sec} = 2832 \text{rev/min} \]

**Determine Efficiency**

Efficiency = \[ \frac{\text{change in speed}}{\text{original speed}} = \frac{4000 - 2832}{4000} = 0.292 \text{ or } 29.2\% \text{ speed reduction} \]

A loss of efficiency of 29.2% in 1 hour is unacceptable. In 3 hours the rotor would have almost stopped rotating. Since the fluid friction is the largest contributor there must be a further reduction in the skin friction.

**FULLY STRESSED DISK DESIGN**

- Concept: Vary the cross section of the disk so that the stress due to rotation remains constant across the radius.
- For a solid disk, theory gives the height as:

\[ Z = Z_0 e^{-\rho \omega^2 r^2 / 2\sigma} \]

- Theoretical disk should have an infinite outer radius
  - thickness tapers down to zero
- Practical disks will have a finite outer radius
  - where the thickness is finite
- For a disk of this shape the moment of inertia is given by:

\[ I = A(1 - e^{-B(B + 1)}) \]

\[ A = \frac{4\pi z_0 \sigma^2}{\rho \omega^2} \quad B = \frac{\rho \omega^2 R_o^2}{2\sigma} \]

\[ R_o = \text{Outer radius} \]